

EXTENSIONS AND APPLICATIONS
OF PRINCIPAL-AGENT PROBLEMS

A thesis
submitted in fulfilment
of the requirements for the
Doctor of Philosophy
in Economics
in the
University of Canterbury
by
Ivette Jans

University of Canterbury

1989

HB
145
J35
1989

DEDICATION

This thesis is dedicated to my family.

ACKNOWLEDGEMENTS

I would like to express special thanks to Richard Manning for his invaluable guidance and comments, encouragement and patience throughout the preparation of this thesis.

Thanks are due to Charles Kahn, Richard Dusansky and Don Gordon for useful suggestions.

Financial assistance from the Reserve Bank of New Zealand is also acknowledged.

CONTENTS

CHAPTER		PAGE
	Abstract	1
1.	Introduction	3
2.	Comparative statics of a principal-agent model .	20
3.	The principal's actions as an incentive device .	42
4.	The core of an economy with moral hazard	73
5.	Incentives and insurance in a partial equilibrium education model	114
6.	Incentives in a general equilibrium education model	149
7.	Summary, concluding remarks and suggestions for future research	179
	Bibliography	183

ABSTRACT

This thesis extends principal-agent models with hidden actions, and uses those models to gain insight into issues in education.

Chapter 2 gives a comparative statics analysis of a conventional model with a single principal and agent. It describes the effect on contracts of changes in the outcome of the principal-agent relationship, stating the results in the form of Slutsky equations.

Continuing with the same model, Chapter 3 allows the principal to choose an action and shows that this action can motivate the agent, and thus act as an incentive device. The model is extended further in Chapter 4, which allows agents to bargain with the principal over the outcome. In this form the model is an extension of a commodity exchange model with uncertain endowments. Using the core as a solution concept, contracts which survive bargaining among agents are derived.

Chapters 5 and 6 consider moral hazard in education. In Chapter 5 the government gives loans to students, and structures loan repayments so that students' future income is to some extent insured. The government chooses the optimal level of insurance, given the possibility of shirking by students. In Chapter 6 the government hires educators, and chooses the optimal compensation of educators, given that they

have an incentive to shirk.

Both models extend principal-agent theory. In Chapter 5 students choose the length of education, so that the length of the principal-agent relationship is endogenous. This may affect the optimal insurance of students' future earnings. In Chapter 6 the number of educators and the skilled wage are endogenous. In terms of principal-agency, optimal contracts are derived in a general equilibrium model in which the number of agents hired by the principal, and their opportunity cost are endogenous.

CHAPTER 1

INTRODUCTION

1.1 EXAMPLES OF MORAL HAZARD

The issue of moral hazard first arose in the insurance literature. Its essence is that insurance induces individuals to take less care in avoiding adverse events, when insurance companies cannot monitor individuals' behavior (for example, Arrow, 1963, Pauly, 1968 and 1974, and Spence and Zeckhauser, 1971). The literature observes that, to alleviate moral hazard, insurance contracts typically exhibit some form of liability rule (such as deductibles or coinsurance), exposing the insured to part of the damages from the occurrence of an insured event.

It has since been noted that insurance is provided not only by insurance companies, but is implicit in many economic arrangements, which are therefore also subject to moral hazard. For instance, fixed wage contracts insure workers against profit fluctuations, inducing them to reduce effort (Stiglitz, 1975, and, where the firm is a landlord and the worker a farmer, Stiglitz, 1974); cost plus contracts remove the incentives to a contractor to avoid costs (Weitzman, 1980, and

McAfee and McMillan, 1986); fixed salary contracts to company managers remove the incentive to maximize the value of the firm (Jensen and Meckling, 1976); insulation from constituents induces elected officials to pursue private goals (McGuire and Ohsfeldt, 1989); borrowers, who do not face the full cost of an unsuccessful investment, overinvest in risky projects (Stiglitz and Weiss, 1981), and product warranties provided by sellers induce less care on the part of buyers (Emons, 1988).¹

Attempts to alleviate moral hazard in these circumstances explains the existence of such phenomena as piece rates, sharecropping, cost plus incentive fee contracts, profit-sharing, financial disclosure laws, credit-rationing, and indemnities in warranties.

Relationships of this nature have come to be known as "principal-agent problems with hidden actions".² The basic problem has been analyzed formally by, among others, Ross (1973) and Holmström (1979),³ and has been surveyed by MacDonald (1984) and Arrow (1985).

1.2 THE BASIC MODEL

The fundamental structure of the model is as follows. A principal and agent are involved in a relationship yielding a range of possible outcomes w_1, \dots, w_n (ranked from least to most preferred), depending on which of n states of nature is

realized.⁴ The principal lays claim to the realized outcome, and makes a payment to the agent. For instance, if the j th state occurs, the agent receives c_j , leaving the principal with $w_j - c_j$. The probability that the j th state occurs is π_j , and depends on an action α taken by the agent in return for the payment. The better the action, the further the probability distribution shifts towards preferred outcomes. While the principal's utility is not directly affected by the action, it generates disutility to the agent. This is a source of conflict between the principal and agent, the principal preferring a more costly action than the agent.

The principal and agent are both expected utility maximizers. If V and U denote their respective state-independent utility functions over income, and if U_0 denotes the disutility to the agent from the action, then the expected utility from a principal-agent contract is

$$EV = \sum_j \pi_j(\alpha) V(w_j - c_j)$$

for the principal, and

$$EU = \sum_j \pi_j(\alpha) U(c_j) - U_0(\alpha)$$

for the agent.⁵ It is assumed that V and U are monotone increasing and concave, and that U_0 is linearly increasing.⁶

Implicit in the principal-agent relationship is that the two parties wish to come to some arrangement. Thus, the principal guarantees that any contract generates the agent's reservation expected utility level (\bar{U}). Formally, the principal faces the participation constraint

$$(1.2.1) \quad EU \geq \bar{U}$$

The principal may face a further constraint, depending on the ability to effectively control the agent's actions. If the principal can dictate actions, then the principal's problem amounts to choosing a compensation scheme $(\hat{c}_1, \dots, \hat{c}_n)$ and action $\hat{\alpha}$ solving

$$(1.2.2) \quad \text{Maximize } EV \quad \text{subject to } (1.2.1)^7 \\ c_1, \dots, c_n, \alpha$$

If, on the other hand, effective control lies with the agent, then the set of actions available to the principal is limited to those which are compatible with the agent's choice.⁸ Formally, any action chosen by the principal must satisfy the following incentive-compatibility constraint:

$$(1.2.3) \quad \alpha \in \arg\max_{\alpha'} EU(c_1, \dots, c_n, \alpha')^9$$

The maximization problem faced by the principal is to choose (c_1^*, \dots, c_n^*) and α^* which solve

(1.2.4) Maximize EV subject to (1.2.1) and (1.2.3)
 c_1, \dots, c_n, α

Prior to any agreement, the principal and agent have knowledge of the possible outcomes, the probability distribution over the outcomes, and the form of the utility functions. Before observing the state of nature, they sign a contract - $(\hat{c}_1, \dots, \hat{c}_n, \hat{\alpha})$ if the principal effectively controls the action; (c_1^*, \dots, c_n^*) otherwise - upon which the agent takes an action. An outcome then occurs, and the corresponding contracted payment is made to the agent.

The standard results from this model are that, in the absence of moral hazard, the optimal contract exhibits perfect risk-sharing between the principal and agent, while, in the presence of moral hazard, the optimal contract sacrifices some risk-sharing opportunities in order to give the agent incentives to choose better actions. Specifically, the agent receives a higher payment the better is the realized outcome. The rationalization is that a better outcome suggests that a better action has been chosen.

1.3 EXTENSIONS OF THE BASIC MODEL

The model has been extended in various directions. For instance, Harris and Raviv (1979) distinguish the above situation from the case where the agent observes the state of

nature before taking an action. Sappington (1984) considers a case where the agent's precontractual knowledge is better than that of the principal. Harris and Raviv (1979), Holmström (1979) and Gjesdal (1982) assume that the principal can observe a variable which contains more information about the action than does the observed outcome. Such monitors help alleviate moral hazard.

Many principal-agent models contain more than one principal or agent. Holmström (1982) analyzes incentives in team production where the principal is unable to attribute a joint outcome to individuals. This literature further observes that when there is more than one agent, the principal can provide incentives by letting the agents compete against one another for given prizes. Such tournaments have been modelled by Lazaer and Rosen (1981), Green and Stokey (1983), and Nalebuff and Stiglitz (1983).¹⁰ Models containing multiple principals with conflicting interests in the actions of an agent are studied by Bernheim and Whinston (1986).

Entirely different incentive devices are analyzed by Radner (1981, 1986), Hart (1983), and Cantor (1988). Radner observes that when the principal and agent contract for more than one period, shirking by the agent may be detected with time, which acts as an incentive to avoid shirking. Hart formalizes the idea that the market mechanism works as an incentive scheme. Cantor shows that when the principal is

unable to base contracts on the outcome, the agent can be motivated by forcing recontracting at predetermined intervals.

Models with double moral hazard are studied by Carmichael (1983), Cooper and Ross (1985) and Emons (1988). They allow the principal to choose an action influencing the expected outcome, and show the implications for contracts when incentives must be created for the principal as well as the agent.

1.4 NEW ASPECTS OF THE PRINCIPAL-AGENT PROBLEM

A criticism of the literature is that it ignores many general equilibrium aspects of the principal-agent relationship.¹¹ For instance, there is no analysis of why the principal and agent find themselves in their respective roles; given their roles, there is little analysis of the circumstances under which it is optimal for both parties to commit themselves to writing a contract (the participation constraint makes any contract acceptable to the agent, while the principal's reservation utility is generally ignored); and the duration of the principal-agent relationship as well as the number of contracted agents are exogenously given. Chapters 4-6 below address these issues in the context of a commodity exchange model with uncertainty, and two education models.

Chapters 2 and 3 are more immediate extensions of the

conventional model. Thus, chapter 2 gives some comparative statics results for the models in (1.2.2) and (1.2.4), describing the effect on the optimal contract of outcome changes in the absence of and presence of moral hazard. The results are stated in the form of Slutsky equations.

Chapter 3 takes off from the model of Carmichael (1983), and is the first step towards the model of chapter 4. It allows the principal to choose an action, β , influencing the expected outcome (that is, $\pi = \pi(\alpha, \beta)$) and so removes one of the asymmetries of the principal-agent problem. The main difference between the existing models, allowing actions by the principal, and the model in Chapter 3 is that the latter allows the principal to explicitly take into account the effect of his own actions on the agent's action. Given positive interdependence between the actions ($\pi^{\alpha\beta} > 0$), better actions by the principal induce better actions by the agent. Thus, the principal's actions are used as an incentive mechanism, as well as a productive input. Chapters 4 and 5 describe models in which π may depend on more than one variable.

The situations discussed in chapter 3 follow the literature in assigning all bargaining power to the principal. When this asymmetry is removed, the principal-agent problem becomes a natural extension of the conventional commodity exchange model with uncertainty. Chapter 4 describes a multi-agent exchange model with uncertain endowments, and

extends this model by allowing each agent to choose an action which influences the probability distribution over the endowments. The model is a general equilibrium version of the models in (1.2.2) and (1.2.4). By eliminating the participation constraint, agents are no longer guaranteed their opportunity cost under exchange. On the other hand, each may achieve more than his opportunity cost through bargaining. The principal-agent relationship is now symmetric - each party has an explicitly defined opportunity cost, has the ability to bargain, and is a potential source as well as victim of moral hazard.

The possibility of bargaining among agents raises the question of what conditions (regarding the nature of bargaining and the environment) will generate acceptable contracts. The usual formulation of the principal-agent model is unable to answer that question, because it assumes, implicitly, that the principal and agent have agreed to cooperate (at issue are the terms of the cooperation). The concept of the core is used to address the issue.

The model is not unlike the multi-agent model of Holmström (1982) in which members of a team contribute effort towards a common output. Holmström, however, does not consider whether the contracts in question are also in the core.

A different interpretation of the model considers the actions of agents as public goods (Holmström (1982) and Radner

(1986) point out the free-rider problem in models where several agents contribute to a common output). Given that each action affects a common probability distribution, every agent benefits fully from, and cannot be excluded from the action. Hence, the action is like a pure public good. In this sense, the analysis is part of the literature on the core of mixed (private and public) goods economies. Initiated, among others, by Shapley and Shubik (1969) in the context of externalities, and Foley (1970) for the case of pure public goods, few general results are available about the nature of the core. The source of the problem is the difficulty of coalitions in becoming independent of non-members (the complementary coalition) who supply public goods. The literature has therefore restricted attention to special types of public goods economies. For example, Pauly (1967) and Wooders (1978) assume away interaction among coalitions by considering local public goods economies; Rosenthal (1971) and Richter (1974) impose rationality conditions upon the complementary coalition, reducing its ability to upset a given coalition; Telser (1982) confers the right to public good supply decisions upon majority coalitions.

The analysis below is in the spirit of such approaches. Although actions are like pure public goods in the sense described above, there are limitations on the set of feasible actions. Further, restrictions are imposed which reduce the interdependence among agents, and the desire or ability of

agents to upset given coalitions.

MacDonald (1984) suggests a further connection with an established approach, namely the game-theoretic literature on revelation mechanisms (for example, Hurwicz, 1972, Townsend, 1979, Palfrey and Srivastava, 1986 and 1989¹²). In the context of an exchange model with uncertainty and private information about agents' characteristics, the problem is to design mechanisms which elicit truthful messages about the characteristic. The model below, in which agents choose actions, is a variation - rather than elicit truthful messages, agents wish to induce appropriate actions. This proves to be a major difference in the analysis of the core - in the private information models, the message of agents who do not belong to a coalition is of no importance to the coalition, so that the interdependence problem discussed above does not exist.

A feature of the commodity exchange model is that the duration of the principal-agent relationship as well as the number of contracted agents are fixed.¹³ In practice, principal-agent relationships may affect some, but not all periods of an agent's life, or some, but not all sectors of the economy. This raises questions involving the optimal duration and optimal extent of activities which are subject to moral hazard, and introduces feedback effects for which the exchange model (and hence special cases such as (1.2.2) and (1.2.4)) is a poor framework. For instance, the individual's work-leisure

choice determines the duration of the firm-worker relationship. Similarly, moral hazard may be a serious problem in some firms, while not in others. The individual's work-leisure choice, and the choice of entrepreneurs (or central planner in a non-market economy) among alternative activities, is likely to depend on the existence of moral hazard.

Chapters 5 and 6 are initial attempts to address these issues. Both are in the context of human capital formation - in chapter 5 the duration of principal-agent contracts is endogenous; in chapter 6 the number of contracted agents is endogenous.

The model in chapter 5 is based on the models of Levhari and Weiss (1974) and Eaton and Rosen (1980), in which individuals choose the amount of education to maximize lifetime utility. Its starting point is the observation by Eaton and Rosen that, if the return to education is uncertain, the tax system can provide insurance. Unlike their model, the cause of uncertainty is made explicit (individuals may graduate or fail), and the uncertainty is endogenous (the chances of graduating depend on individuals' effort and the length of the education program). There are no private markets for capital or insurance, so that the government finances education by way of loans, and structures loan repayments to insure individuals against failure. As long as there is some insurance¹⁴ (that is, graduates repay more than failures) student effort

generates a positive externality by raising the expected loan repayment. Hence, there is a principal-agent problem.¹⁵

The model is more general than the basic single agent model described earlier, because individuals, in addition to choosing the level of effort, choose the length of the education program. Thus, the duration of the principal-agent contract is endogenous. The main issues are whether the usual result of incomplete risk-sharing when there is moral hazard still holds, and whether the optimal level of insurance is affected when the duration of the principal-agent relationship (as well as the agent's action) affects the expected outcome.

A shortcoming of the model is that it describes a partial equilibrium. Specifically, it is always optimal to acquire an education (in terms of the principal-agent literature, since all individuals choose an education, the number of agents contracted by the principal is exogenous); and there is an incomplete link between the relative amount of skill and the relative return to skill.

Manning (1975, 1976, 1985) describes a general equilibrium model which overcomes both problems. Labor is used in the production of a good, but it is optimal to educate some individuals before they enter the workforce. The optimal size of the education sector is determined by a welfare-maximizing planner. Unlike the model in Chapter 5, the model in Chapter 6 recognizes that the cost of education involves the cost of

hiring educators, as well as the cost of students' foregone earnings. Further, educators may have an incentive to shirk. If other factors, such as the quality of students, also influence students' performance, and the planner cannot observe those factors nor the activities of educators, then the educators may shirk without detection. To alleviate such moral hazard the planner offers performance-dependent contracts, penalizing educators for observed poor class performance. The chapter discusses properties of such contracts.

From the point of view of principal-agency, the planner is like a principal and the educators act as agents.¹⁶ The model is novel, because in order to derive the optimal contract, the planner must determine the optimal number of educators and their opportunity cost.

1.5 NOTES

1. See also Zeckhauser (1970), Kihlstrom (1971), Mirrlees (1976), Harris and Raviv (1978), Hurwicz and Shapiro (1978), Shavell (1979a, 1979b and 1982), and Gjesdal (1988).
2. Principal-agent models with hidden information (that is, adverse selection models) are not considered.
3. See also Harris and Raviv (1979), Shavell (1979b), and Grossman and Hart (1983).
4. The model has been analyzed with discrete as well as continuous outcomes. The discrete version is given here, because in the following chapters (except Chapter 2) outcomes are limited to two possibilities.
5. To avoid technical problems, much of the literature assumes that the agent's utility function is separable in income and effort.
6. Results are available for more general formulations (for instance, some assume strictly convex U_0), but a constant marginal disutility from the action suffices in the following analyses.
7. The optimal action is implemented by way of a "forcing" contract (for example, Harris and Raviv, 1979) the form of which is given in Appendix 2 of Chapter 4, section 4.13.
8. That is, the principal can only "choose" actions which, given the compensation scheme, are willingly chosen by the

agent, because they maximize the agent's expected utility. If only one action maximizes the agent's expected utility, then the principal in essence has no choice.

9. The early literature replaces this constraint by the first-order condition for an interior maximum of the agent's expected utility, namely $\partial EU/\partial \alpha = 0$. Mirrlees (1975) observes that this condition is neither necessary nor sufficient for the incentive-compatibility constraint to be satisfied. Mirrlees and subsequently Rogerson (1985), Brown and others (1986), and Jewitt (1988) derive conditions under which the "first-order approach" (that is, replacing (1.2.3) with $\partial EU/\partial \alpha = 0$) is valid. The models below assume that the Mirrlees-Rogerson conditions hold. These are the monotone likelihood ratio condition (MLRC):

$$w_j \leq w_i \Rightarrow G(\alpha|w_i) \leq G(\alpha|w_j) \quad \forall i, j \quad \forall \alpha$$

where $G(\alpha|w_j)$ is the posterior probability, given that outcome w_j is observed; and the convexity of the distribution function condition (CDFC):

$$F_j'(\alpha) \geq 0 \quad \forall j, \quad \forall \alpha$$

where $F_j = \sum_{h=1}^j \pi_h(\alpha)$. Intuitively, the MLRC states that a higher outcome is evidence that the agent has worked harder. In this sense effort is productive. The CDFC states that effort is

subject to diminishing returns.

10. See also Mookherjee (1984), Malcomson (1984, 1986), Rosen (1986) and Bhattacharya and Guasch (1988).

11. Some of these are observed in McDonald's survey.

12. See also Dasgupta and others (1979), Myerson (1979), and Postlewaite and Schmeidler (1986).

13. If there is no core, both are zero; if there is a core, then the duration of any contract equals the timeframe of the model, and involves all agents.

14. It is shown that some insurance is optimal.

15. Although the government writes contracts with many students, this is a multi-agent model only in the trivial sense. Each student writes a contract with the government independently of other students.

16. As in Chapter 5, this is a multi-agent model only in the trivial sense. Each educator contracts with the government independently of other educators.

CHAPTER 2

COMPARATIVE STATICS OF A PRINCIPAL-AGENT MODEL

2.1 INTRODUCTION

McDonald's 1984 survey of the principal-agent literature points out that the comparative statics of the agency problem are yet to be fully worked out. The objective below is to provide some of that analysis for the principal-agent models in (1.2.2) and (1.2.4). Effects on the optimal principal-agent contract of changes in the outcomes faced by the principal are described. The results are given in the form of Slutsky equations, obtained by applying the generalized Slutsky system of Chichilnisky and Kalman (1977, 1978). In their 1977 paper, the authors derive Slutsky equations for a very broad class of constrained maximization problems, including problems with parameters entering the objective function, and multiple constraints, such as the principal-agent problem.

Section 2.2 states the generalized Slutsky system, and reexpresses it in the form of elasticity relationships. Sections 2.3 and 2.4 impose the restrictions implied by the principal-agent models in (1.2.2) and (1.2.4), respectively. The Slutsky equations in elasticity form allow some observations about the responsiveness of contracts to outcome

changes. Section 2.5 concludes.

2.2 THE SLUTSKY EQUATIONS

Chichilnisky and Kalman derive a Slutsky-type decomposition for a general constrained maximization problem of the form

$$(2.2.1) \quad \begin{array}{ll} \text{Max} & f(x, a) \\ & x \\ \text{s.t.} & g(x, a) = b \end{array}$$

where f is an objective function, g is a p -vector of constraint functions, x is an n -vector of instruments, a is an m -vector of parameters, and b is a p -vector of constraint parameters.¹ In the well-known Slutsky equation of the consumer choice model, changes in income hold constant the consumer's utility, to derive the substitution effects of price changes. As Chichilnisky and Kalman point out, in model (2.2.1), any one of the constraint parameters, as well as linear combinations thereof, can be used to hold constant the value of f in response to a change in one of the parameters, a_1, \dots, a_m . A Slutsky-type decomposition exists for each method of holding constant the value of f . If the parameter b_k is used to make such a compensation, then the authors derive the following Slutsky-type decomposition near the optimal value of x :

$$(2.2.2)^{2,3} \quad \frac{\partial x}{\partial a} + \frac{\partial x}{\partial b} \frac{\partial g}{\partial a} = \frac{\partial x}{\partial a} \Big|_f + \frac{\partial x}{\partial b} \left[\phi \frac{\partial g}{\partial a} - \mu \frac{\partial f}{\partial a} \right]$$

where $\frac{\partial x_i}{\partial a_j} \Big|_f$ denotes the effect of a change in a_j on x_i ,

holding f constant;

$$\phi = \begin{bmatrix} 1 & & & & & & & \\ & \cdot & & & & & & \\ & & \cdot & & & & & \\ & & & 1 & & & & \\ -\frac{\lambda_1}{\lambda_k} & \cdot & \cdot & \cdot & -\frac{\lambda_{k-1}}{\lambda_k} & 0 & -\frac{\lambda_{k+1}}{\lambda_k} & \cdot & \cdot & \cdot & -\frac{\lambda_p}{\lambda_k} \\ & & & & & 1 & & & & & \\ & & & & & & \cdot & & & & \\ & & & & & & & \cdot & & & \\ & & & & & & & & 1 & & \end{bmatrix}$$

with 0 in all unfilled places;

$$\mu = \begin{bmatrix} 0 & \cdot & \cdot & 0 & \frac{1}{\lambda_k} & 0 & \cdot & \cdot & 0 \end{bmatrix}$$

and $\lambda = (\lambda_1, \dots, \lambda_p)$ is the p -vector of Lagrange multipliers in

$$L(x, \lambda, a, b) = f(x, a) + \lambda (g(x, a) - b)'$$

The Slutsky system in (2.2.2) relates the effects on x of changes in the parameters and constraint parameters. It states that if a programming problem can be described by (2.2.1), then the effect on x_i of a change in a_j can be decomposed into

-the effect on x_i of a change in b_k , calculated to hold f

constant; and

-the effect on x_i of a compensated change in a_j , where "compensated" denotes that f is held constant by the above change in b_k .

The Slutsky equations can be reexpressed in elasticity form. Since the choice of b_k affects the values of the derivatives below, each elasticity is conditional on that choice.

$$e_{ij} = \frac{a_j}{x_i} \frac{\partial x_i}{\partial a_j} \quad i=1, \dots, n \quad j=1, \dots, m$$

$$n_{ij} = \frac{a_j}{x_i} \frac{\partial x_i}{\partial a_j} \Big|_f \quad i=1, \dots, n \quad j=1, \dots, m$$

$$u_{ik} = \frac{b_k}{x_i} \frac{\partial x_i}{\partial b_k} \quad i=1, \dots, n \quad k=1, \dots, p$$

$$s_{jk}^h = \frac{\lambda_h a_j \frac{\partial g_h}{\partial a_j}}{\lambda_k b_k} \quad j=1, \dots, m \quad h, k=1, \dots, p$$

$$s_{jk}^{p+1} = \frac{a_j \frac{\partial f}{\partial a_j}}{\lambda_k b_k} \quad j=1, \dots, m \quad k=1, \dots, p$$

The first three terms measure the elasticity of x_i with respect to, respectively, a change in a_j , a change in a_j holding f constant, and a change in b_k . s_{jk}^h and s_{jk}^{p+1} measure the value

(in units of the k th constraint parameter) of a_j , generated by its effect on, respectively, the h th constraint function and the objective function.

A typical entry in (2.2.2) (with $(\partial x/\partial b)(\partial g/\partial a)$ taken to the right-hand side) is

$$(2.2.3) \quad \frac{\partial x_i}{\partial a_j} = \frac{\partial x_i}{\partial a_j} \Big|_f + \frac{\partial x_i}{\partial b_k} \left[\sum_{h=1}^p \left\{ -\frac{\lambda_h}{\lambda_k} \frac{\partial g_h}{\partial a_j} \right\} - \frac{1}{\lambda_k} \frac{\partial f}{\partial a_j} \right]$$

$$i=1, \dots, n \quad j=1, \dots, m \quad k=1, \dots, p$$

Multiplying (2.2.3) by $\frac{a_j}{x_i}$ gives

$$(2.2.4) \quad e_{ij} = n_{ij} - u_{ik} \left[\sum_{h=1}^{p+1} s_{jk}^h \right]$$

$$i=1, \dots, n \quad j=1, \dots, m \quad k=1, \dots, p$$

2.3 APPLICATION: A PRINCIPAL-AGENT MODEL WITH ENFORCEABLE ACTIONS

Let $m=n-1$ and $p=1$. Let a_i $i=1, \dots, n-1$ be the outcome in state i of a principal-agent relationship with $a_1 < \dots < a_{n-1}$; let x_i $i=1, \dots, n-1$ be the payment from principal to agent in state i ; and let x_n be the agent's effort. Define

$$f(x, a) = \sum_{h=1}^{n-1} \pi_h(x_n) V(a_h - x_h)$$

$$g_1(x) = \sum_{h=1}^{n-1} (\pi_h(x_n) U(x_h)) - x_n$$

where π_h is the probability that a_h occurs, V and U are the utility functions of the principal and agent, V is linearly⁴ increasing in $a_h - x_h$; U is strictly concave and increasing in x_h ; and the Mirrlees-Rogerson conditions validating the first-order approach are satisfied.⁵ Let b_1 denote the agent's reservation price (that is, the minimum expected utility level required to induce the agent to accept a contract from the principal). If the principal controls the payments and the action of the agent, then (assuming that the agent is paid exactly the reservation price), the principal-agent problem is

$$(2.3.1) \quad \begin{array}{ll} \text{Max} & f(x, a) \\ & x \end{array}$$

$$\text{s.t.} \quad g_1(x) = b_1$$

Problem (2.3.1) is a special case of problem (2.2.1), so that application of (2.2.3) and (2.2.4) gives a relationship between the effect on the principal-agent contract of changes in outcomes and changes in the agent's reservation price. Using

$$\frac{\partial g_1}{\partial a_j} = 0 \quad \text{and} \quad \frac{\partial f}{\partial a_j} = \pi_j(x_n) V'(a_j - x_j) \quad j=1, \dots, n-1 \text{ gives}$$

$$(2.3.2) \quad \frac{\partial x_i}{\partial a_j} = \frac{\partial x_i}{\partial a_j} \Big|_f - \frac{\partial x_i}{\partial b_1} \frac{1}{\lambda_1} \pi_j(x_n) V'(a_j - x_j)$$

$$i=1,\dots,n \quad j=1,\dots,n-1$$

or in elasticity form

$$(2.3.3) \quad e_{ij} = n_{ij} - u_{i1}s_{j1} \quad i=1,\dots,n \quad j=1,\dots,n-1$$

where

$$s_{j1} = \frac{\pi_j(x_n) a_j V'(a_j - x_j)}{\lambda_1 b_1}$$

To explain the Slutsky equations, Table 1 (which is derived in Appendix 1, Section 2.6) gives the signs of the derivatives in (2.3.2). These depend on whether a favorable (that is, relatively large) or unfavorable outcome has changed. The notion of a favorable outcome is made precise in:

Definition 1

a_j is a favorable (an unfavorable) outcome iff $\pi_j' \geq (<) 0$
 $j=1,\dots,n-1$.

Given the agent's action x_n , the monotone likelihood ratio condition, identified by Mirrlees and Rogerson, defines $r \in (1, n-1]$ such that $\pi_j' \geq 0$ if and only if $j \geq r$.⁶ Definition 1 then states that a_j is favorable if and only if $j \geq r$. Appendix 1 derives the results that an increase in favorable outcomes leads the principal to specify larger payments and higher effort, while the effect of increases in unfavorable outcomes is the opposite. Further, increases in the agent's reservation

price are met by a combination of higher payments and lower effort:

Table 1⁷

	a_j $j < r$	a_j $r \leq j$	b_1
x_i $i \leq n-1$	-	+	+
x_n	-	+	-

The Slutsky equations can be explained as follows. The principal has to decide on an optimal tradeoff between effort and payments.⁸ The higher the level of effort, the larger the payments necessary to compensate the agent. The optimal tradeoff depends (among other things) on the size of the outcomes. The left-hand side of the Slutsky equations show how effort and payments actually change if the outcomes change. The right-hand side decomposes this into two effects. An increase in any outcome makes the principal, who appropriates the increase, better off. Suppose that there is no such utility effect. Specifically, suppose that the agent's reservation price increases sufficiently to hold constant the principal's expected utility. Then the effect of the outcome change on the terms of the contract is a pure substitution effect, and is measured by the first term in each Slutsky equation. Now consider the utility effect. The increase in

expected utility experienced by the principal is equivalent to a fall in the agent's reservation price. The resulting effect on the contract is measured by the second term in each Slutsky equation.

A use of the Slutsky equation is that knowledge of the responsiveness of contracts to changes in the agent's reservation price gives an idea of the responsiveness of contracts to outcome changes. Table 1 implies that $e_{ij} > 0$ iff $r \leq j \quad \forall i=1, \dots, n \quad j=1, \dots, n-1$; $u_{i1} > 0 \quad i=1, \dots, n-1$, $u_{n1} < 0$, and $s_{j1} > 0 \quad j=1, \dots, n-1$. The responsiveness of the contract to outcome changes is measured by the absolute value of e_{ij} . The equations in (2.3.3) state that, the smaller the elasticity of payments with respect to the agent's reservation price (the smaller is $u_{i1} \quad i=1, \dots, n-1$), and the larger the elasticity of effort with respect to the agent's reservation price (the larger is $-u_{n1}$), the larger is the elasticity of payments and effort with respect to changes in favorable outcomes (the larger is $e_{ij} \quad i=1, \dots, n \quad j \geq r$). This result can be explained in terms of the substitution and utility effects identified above.

Suppose that a favorable outcome increases. To make that outcome more likely, the principal substitutes towards larger payments to obtain higher effort. The increase in the outcome has a utility effect similar to a fall in the agent's reservation price, namely a fall in payments and increase in effort (see Table 1). Since the utility effect on payments

opposes the substitution effect, whereas the utility effect on effort reinforces the substitution effect, the less responsive are payments, and the more responsive is effort, to changes in the agent's reservation price, the more responsive are payments and effort to the given outcome change.

On the other hand, an increase in an unfavorable outcome causes the principal to substitute towards lower payments and lower effort. In this case the utility effect on payments (effort) reinforces (opposes) the substitution effect. Hence, equation (2.3.3) further predicts that the smaller (greater) the elasticity of payments (effort) with respect to the agent's reservation price, the smaller the elasticity of payments and effort with respect to unfavorable outcome changes (the smaller is $-e_{ij}$ $i=1, \dots, n-1$ $j < r$).

2.4 APPLICATION: A PRINCIPAL-AGENT MODEL WITH UNENFORCEABLE ACTIONS

If the principal controls the payments and the agent controls the action, then, assuming binding constraints, the principal's problem is

$$\begin{aligned}
 (2.4.1) \quad & \text{Max}_x \quad f(x, a) \\
 & \text{s.t.} \quad g_k(x) = b_k \quad k=1, 2
 \end{aligned}$$

where f and g_1 are defined as in section 2.3 and constrained by the conditions outlined there; the condition that $-\pi_1'/\pi_1'$ is relatively large is imposed;⁹ $b_2=0$ and

$$g_2(x) = \frac{\partial g_1}{\partial x_n} = \sum_{h=1}^{n-1} \pi_h'(x_n) U(x_h) - 1.$$

The second constraint is the incentive-compatibility constraint, given validity of the first-order approach (see note 9, Chapter 1).

In this problem there are two constraint parameters, namely b_1 , measuring the agent's reservation price, and b_2 , measuring the change in the agent's expected utility, when there is an infinitesimal change in effort. A change in b_2 amounts to a change in the form of the agent's expected utility function. Hence, it is more natural to hold f constant by changing b_1 , in response to a change in the outcomes.

Application of (2.2.3) and (2.2.4) again gives the Slutsky equations in (2.3.2) and (2.3.3). In this case they hold near the second-best optimum. To simplify calculations, Table 2 (derived in Appendix 2, Section 2.7) reports the signs of the derivatives in (2.3.2) when there are only two possible outcomes.¹⁰ An increase in the favorable outcome a_2 (as defined in section 2.3) causes an increase in effort and payment in the favorable state, and a decrease in payment in

the unfavorable state. The effect is opposite for an increase in the unfavorable outcome a_1 . An increase in the agent's reservation price leads to higher payments and higher effort.

Table 2

	a_1	a_2	b_1
x_1	+	-	+
x_2	-	+	+
x_3	-	+	+

Table 2 implies that $e_{ij} > 0$ iff $i=j$ $i,j=1,2$, $e_{31} < 0$, $e_{32} > 0$, $u_{i1} > 0$ $i=1,2,3$, and $s_{j1} > 0$ $j=1,2$. Equation (2.3.3) therefore predicts that the larger (smaller) the elasticity of payments in the unfavorable (favorable) state, and the smaller the elasticity of effort, all with respect to changes in the agent's reservation price (the larger is u_{11} , and the smaller are u_{21} and u_{31}), the larger the elasticity of payments and effort with respect to changes in the favorable outcome (the larger are e_{12} , e_{22} , and e_{32}).

For instance, suppose that the favorable outcome increases. The substitution effect is that the principal wants to make that outcome more likely, which requires a higher effort. Unlike problem (2.3.1), higher effort cannot be enforced - effort can be affected only by creating incentives to the agent via the payments. Thus, the principal raises

(lowers) payment in the event of a large (small) outcome. The utility effect is that the principal has a higher expected utility. This is like a fall in the agent's reservation price, which would induce lower payments and effort (see Table 2). Since the utility effect on payments in state 1 reinforces the substitution effect, while the utility effect on payments in state 2 and on effort opposes the substitution effect, greater responsiveness of state 1 payments, and smaller responsiveness of state 2 payments and effort to changes in the agent's reservation price imply greater responsiveness of the contract to changes in the favorable outcome.

An analogous argument explains why greater responsiveness of payments in state 1, and smaller responsiveness of payments in state 2 as well as effort, to changes in the agent's reservation price are associated with smaller responsiveness of the contract to changes in the unfavorable outcome.

2.5 CONCLUSION

Chichilnisky and Kalman's system of generalized Slutsky equations gives insight into the response of principal-agent contracts to exogenous disturbances in the outcomes. Rewriting the equations in terms of elasticities, gives a relationship between the responsiveness of contracts to changes in the

agent's reservation price, and the responsiveness of contracts to outcome changes.

As an example, consider a worker who carries out observable tasks for a firm in return for a fixed wage. The firm's profits, which may turn out high or low, determine the level of wages and effort (that is, whether both will be relatively high or low). Suppose that the firm experiences an increase in the profits that will be realized if business turns out to be good. Suppose further, that the firm has frequently rewritten the contract, due to changes in the worker's opportunity cost. Equation (2.3.3) states that the larger were the proportionate adjustments in wages, and the smaller the proportionate adjustments in effort, the larger will be the proportionate upward adjustment in wages and effort due to the increase in profits.

2.6 APPENDIX 1 - DERIVATION OF TABLE 1¹¹

The first-order conditions of problem (2.3.1) (evaluated at the optimum) are

$$L^i = \pi_i V'(i) - \lambda_1 \pi_i U'(i) = 0 \quad i=1, \dots, n-1$$

$$L^n = \sum_h \pi_h' V(h) + \lambda_1 (\sum_h \pi_h' U(h) - 1) = 0$$

$$L^{\lambda_1} = \sum_h \pi_h U(h) - x_n - b_1 = 0$$

where $V(i) \equiv V(a_i - x_i)$ and $U(i) \equiv U(x_i)$

Total differentiation of the first-order conditions gives the system of equations

$$(2.6.1) \quad \begin{bmatrix} L^{11} & & & & & 0 & L^{1\lambda_1} \\ & \cdot & & 0 & & \cdot & \cdot \\ & & \cdot & & & \cdot & \cdot \\ & & 0 & \cdot & & \cdot & \cdot \\ & & & L^{n-1n-1} & & 0 & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & L^{nn} & L^{n\lambda_1} \\ L^{\lambda_1 1} & \cdot & \cdot & \cdot & \cdot & L^{\lambda_1 n} & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ dx_n \\ d\lambda_1 \end{bmatrix} =$$

$$\begin{bmatrix} & & (0) & & \\ p^{n1} & . & . & . & . & p^{nn-1} & 0 \\ 0 & . & . & . & . & 0 & 1 \end{bmatrix} \begin{bmatrix} da' \\ db_1 \end{bmatrix}$$

where (0) is an $n-1$ by n matrix of zeros and $da=(da_1, \dots, da_{n-1})$

$$L^{ii} = \lambda_1 \pi_i U''(i) < 0 \quad i=1, \dots, n-1$$

$$L^{i\lambda_1} = \pi_i U'(i) > 0 \quad i=1, \dots, n-1$$

$$L^{nn} = \sum_h \pi_h' (V(h) + \lambda_1 U(h)) < 0$$

$$L^{n\lambda_1} = \sum_h \pi_h' U(h) - 1 < 0 \quad \text{using } L^n=0$$

$$P^{nj} = -\pi_j' V'(j) > 0 \quad \text{iff } j < r \quad j=1, \dots, n-1$$

The second-order conditions require that, at the optimum, the coefficient matrix on the left-hand side of (2.6.1) is negative semi-definite. The results in Table 1 follow from the properties of π , V and U , and from the application of Cramer's rule to (2.6.1).

2.7 APPENDIX 2 - DERIVATION OF TABLE 2

The first-order conditions of problem (2.4.1) (evaluated at the optimum) are

$$L^i = -\pi_i V'(i) + (\lambda_1 \pi_i + \lambda_2 \pi_i') U'(i) = 0 \quad i=1,2$$

$$L^3 = \sum_h \pi_h' V(h) + \lambda_1 (\sum_h \pi_h' U(h) - 1) + \lambda_2 \sum_h \pi_h' U(h) = 0$$

$$L^{\lambda_1} = \sum_h \pi_h U(h) - x_3 - b_1 = 0$$

$$L^{\lambda_2} = \sum_h \pi_h' U(h) - 1 = 0$$

Total differentiation of the first-order conditions gives the system of equations

$$(2.7.1) \quad \begin{bmatrix} L^{11} & 0 & L^{13} & L^{1\lambda_1} & L^{1\lambda_2} \\ 0 & L^{22} & L^{23} & L^{2\lambda_1} & L^{2\lambda_2} \\ L^{31} & L^{32} & L^{33} & 0 & L^{3\lambda_2} \\ L^{\lambda_1 1} & L^{\lambda_1 2} & 0 & 0 & 0 \\ L^{\lambda_2 1} & L^{\lambda_2 2} & L^{\lambda_2 3} & 0 & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \\ d\lambda_1 \\ d\lambda_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ p^{31} & p^{32} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} da_1 \\ da_2 \\ db_1 \end{bmatrix}$$

where

$$L^{ii} = (\lambda_1 \pi_i + \lambda_2 \pi_i') U''(i) < 0 \quad \text{using } L^i = 0 \quad i=1,2$$

$$L^{i3} = (-V'(i) + \lambda_1 U'(i)) \pi_i + \lambda_2 \pi_i' U'(i)$$

$$> 0 \quad \text{if } i=1 \text{ and } \frac{\pi_i''}{-\pi_i'} \text{ is relatively large}$$

$$< 0 \quad \text{if } i=2$$

$$L^{i\lambda_1} = \pi_i U'(i) > 0 \quad i=1,2$$

$$L^{i\lambda_2} = \pi_i' U'(i) < (>) 0 \quad \text{if } i=1(2)$$

$$L^{33} = \sum_h \pi_h'' (V(h) + \lambda_1 U(h)) + \lambda_2 \pi_h''' U(h) < 0 \quad (\text{if } \pi_h''' \approx 0)$$

$$L^{3\lambda_2} = \sum_h \pi_h' U(h) < 0$$

The second-order conditions require that, at the optimum, the coefficient matrix on the left-hand side of (2.7.1) is negative

semi-definite. The results in Table 2 follow from the properties of π , V and U , and application of Cramer's rule to (2.7.1).

2.8 NOTES

1. This is the terminology used by Kalman and Intriligator (1973), who were the first to provide a generalized system of Slutsky equations.

2. Notation is as follows. If x is an n -vector and y is an m -vector, then

$$\frac{\partial x}{\partial y} \equiv \begin{bmatrix} \frac{\partial x_1}{\partial y_1} & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{\partial x_1}{\partial y_m} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \frac{\partial x_n}{\partial y_1} & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{\partial x_n}{\partial y_m} \end{bmatrix}$$

x' is the transpose of x .

3. The terms on the left-hand side are observable effects, while the terms on the right-hand side are unobservable effects.

4. If the principal is risk-averse, the Slutsky-type decomposition is complicated by the desire of the principal to find a new way of spreading the risk associated with the outcomes, when one of the outcomes changes. It is difficult to derive general results for this case.

5. These are the monotone likelihood ratio and convexity of the distribution function conditions, which guarantee that the solution to the principal-agent problem in (2.3.1) below

describes an action, which maximizes the agent's expected utility (see note 9, Chapter 1).

6. The monotone likelihood ratio condition can be stated as

$$\frac{\pi'_h(\cdot)}{\pi_h(\cdot)} \leq \frac{\pi'_{h+1}(\cdot)}{\pi_{h+1}(\cdot)} \quad h=1, \dots, n-2 \quad \forall x_n$$

(see Milgrom, 1981). Thus, if $\pi'_i \geq (<) 0$ then $\pi'_j \geq (<) 0$ $\forall j > (<) i$.

(Suppose not. For instance, let $\pi'_i \geq 0$, $\pi'_j < 0$ with $j > i$. Then

$$\frac{\pi'_j}{\pi_j} < \frac{\pi'_i}{\pi_i} \quad \text{which does not satisfy the MLRC.})$$

Define r such that $\pi'_{r-1} < 0$ and $\pi'_r \geq 0$.

7. If the principal is risk-averse, the effect described in note 4 may lead to reversal of some signs.

8. Given risk-neutrality of the principal, payments are equal across states (see, for instance, Harris and Raviv, 1979).

9. This is suggested by the second-order conditions.

Specifically, the second-order conditions hold if $(\pi'_1)^2 - \pi''_1 \leq 0$ (thus, the condition is sufficient). This is satisfied by functions such as $\pi(x_n) = x_n^{\frac{1}{2}}$ $x_n \in (0, 1)$ and $\pi(x_n) = \log x_n$ $x_n \in (1, \exp(1))$.

10. More general results are not available. However, it is not unusual practice to confine nature to two states (for

instance, Shavell, 1979a, and Arnott and Stiglitz, 1988).

11. To make the arguments in the text less cumbersome, it is assumed that $\pi_j' \neq 0 \forall j$.

CHAPTER 3

THE PRINCIPAL'S ACTIONS AS AN INCENTIVE DEVICE

3.1 INTRODUCTION

When the principal as well as the agent can choose an action influencing the expected outcome, there are circumstances in which the principal has at his disposal a new mechanism for creating incentives to the agent. Specifically, if there is positive interdependence between the actions of the players, and the principal perceives that his action can influence the agent, then the principal can use his own action as an incentive device.

Many principal-agent examples involve a principal who, in practice, is likely to be an active contributor to the outcome, rather than a passive observer, whose sole task is to specify a compensation scheme. For instance, in the standard insurance context (see Spence and Zeckhauser, 1971), the expected payout by the insurance company is affected by the preventive care taken by the insured. However, the payout is also affected by information about accident prevention made available by the insurers. Professionals such as doctors and lawyers act as agents for clients whose cooperation influences the success of their cases. In education the success of

students depends not only on their own effort, but also on the input from educators.

The observation that the principal may take an action is not new. Carmichael (1983) studies the firm-worker relationship, in which the firm can influence the level of output by actions such as advertizing. Cooper and Ross (1985), and Emons (1988) discuss a buyer who faces moral hazard on the part of a seller, because the seller chooses an unobservable quality level. They note that the buyer's care of the product also affects its quality. The focus in these papers is on the issue of double moral hazard; that is, the problem that not only the principal is confronted with moral hazard, but so is the agent, since the principal may shirk (for example, the firm may not advertize sufficiently from the point of view of the worker; warranties designed to increase the quality chosen by the seller induce less care on the part of the buyer). This form of double moral hazard is not at issue below. Since, ex ante, the agent is always guaranteed his opportunity cost, it is arguable that, even when the principal chooses actions which cannot be controlled by the agent, the agent is never harmed, and so never faces moral hazard. On the other hand, the agent may be able to control the principal's action, as well as his own. In this sense, the principal may be confronted with double moral hazard on the part of the agent.¹

A question addressed below is how the principal can

motivate the agent to choose proper actions, when the agent controls his own action, the action of the principal, or both. It is observed that a principal who knows how the agent reacts to a contract, will take this information into account when choosing his own action. The principal's action therefore has incentive effects (indirectly contributing to the outcome) as well as a direct effect on the outcome. The existing literature observes that the principal's action contributes to the outcome, but does not discuss its use as an incentive device. Carmichael, Cooper and Ross formulate the problem so that in the first stage of the game the payments are chosen. In the second stage the principal and agent each choose an action under Nash conjectures. Thus, each player takes as given the other's action, and does not perceive its response to his own action. In Emons, there is not enough interdependence among actions to generate the incentive effect identified below.

The analysis is organized as follows. Section 3.2 illustrates the issues by way of an example. The model is set out in section 3.3. Sections 3.4-3.7 characterize the payments and actions under various assumptions about which party has control over the actions. Section 3.8 concludes.

3.2 AN EXAMPLE

Consider a firm which hires a consultant to update its communications system. The project may succeed or fail. The chances that it will succeed improve with greater effort from the consultant, and greater effort (such as cooperation with the consultant) from an operator, who is to use the new system. Further, greater effort from the operator raises the marginal product of the consultant (that is, an extra hour of effort by the consultant produces a larger improvement in the network, the greater the effort from the operator). Suppose that effort generates disutility to the consultant, and that the operator must be compensated by the firm for his effort. Then maximum effort by the consultant or operator is generally not optimal. If the firm has control over the payment made to the consultant, at least four possibilities arise depending on who controls the actions of the consultant and operator:

(3.2.1) The firm's expected utility (net of the consultant's opportunity cost) is highest if it knows as well as the consultant which actions are desirable from its own perspective, and if it can observe those actions, so that they can be written into a contract. Payment to the consultant will entail perfect risk-sharing between the consultant and firm.

(3.2.2) If the firm is able to monitor the effort of the operator, but not that of the consultant (for example, the consultant may have to carry out tasks away from the firm), then the firm faces the possibility of shirking by the consultant. It should use the familiar remedy of paying more to the consultant, the greater the success of the project. The firm can also use the operator to motivate the consultant. By specifying a greater level of effort for the operator (in return for higher wages) the firm raises the marginal product of the consultant, inducing greater effort. Thus, the firm's ability to actively influence the outcome of the project creates a new incentive mechanism. In addition, the greater effort from the operator directly increases the chances of success, and so reduces the adverse effect of the consultant's shirking.

(3.2.3) Suppose that the firm is able to monitor the consultant's effort (once at the firm, the consultant may prefer working to not working, so that the firm need only be sure of the consultant's presence), but does not know what tasks the operator should perform, leaving the more knowledgeable consultant to specify those tasks. Since effort by the operator enhances the probability of a successful project, yet involves no cost to the consultant, the consultant will specify the maximum effort as long as he is paid more when

the project is successful. If the consultant overworks the operator only when that is to his own benefit, the firm can solve this form of moral hazard provided the firm is not too risk-averse. By paying the consultant a fee that is independent of the outcome, the consultant is indifferent to the actions of the operator, and is therefore willing to specify the tasks most appropriate for the firm.

On the other hand, a sufficiently risk-averse firm will transfer some risk to the consultant (offer a higher payment for a successful project) at the cost of inducing the consultant to work the operator as hard as possible. Since any exposure of the consultant to risk motivates the consultant in this way, the firm chooses the payment scheme which perfectly shares risk.

(3.2.4) Now suppose that the firm in (3.2.3) is also unable to monitor the consultant's effort. If the firm bears all of the risk associated with the project, the consultant will choose minimal effort for himself. To avoid this the firm must pay less for an unsuccessful project. Thus, the firm has to trade one form of moral hazard (too little effort by the consultant) against another (too much effort specified by the consultant for the operator). If the consultant and operator are sufficiently productive, the firm will expose the consultant to some risk to elicit above minimal effort, at the cost of having

to provide high compensation to the operator for carrying out the tasks specified by the consultant.

3.3 THE MODEL

Consider the models in (1.2.2) and (1.2.4) with the number of states limited to two. Let π denote the probability that the favorable state (2) occurs, and suppose that this probability is affected by the actions, $\beta \in [\underline{\beta}, \bar{\beta}]$, of the principal, as well as by the actions, $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, of the agent; that is, $\pi = \pi(\alpha, \beta)$. It is assumed that $\pi(\alpha, \underline{\beta}) = \pi(\underline{\alpha}, \beta) = 0$, $\pi^\alpha > 0$, $\pi^\beta > 0$, $\pi^{\alpha\alpha} < 0$, $\pi^{\beta\beta} < 0$, and $\pi^{\alpha\beta} > 0$. In words, if the actions are levels of effort, a necessary condition for a favorable outcome is that minimal effort be avoided; greater effort from either agent or principal increases the probability of the favorable state, but at a diminishing rate; and greater effort from the principal raises the marginal product of the agent (that is, the actions are complements).

Both players dislike effort - the marginal disutility from effort is v for the principal, and u for the agent (v and u are assumed constant). If the state-independent utility functions over the outcome are V for the principal and U for the agent, then the expected utilities are

$$EV = (1-\pi)V(w_1 - c_1) + \pi V(w_2 - c_2) - v\beta$$

for the principal, and

$$EU = (1-\pi)U(c_1) + \pi U(c_2) - u_\alpha$$

for the agent. It is assumed that U and V are monotone increasing, that U is strictly concave, and that V is concave.

The variables which are controlled by the principal are written into a contract before the variables under the control of the agent are chosen by the agent. The principal always controls the terms of compensation, but not necessarily the actions. All choices are made before an outcome is realized.

To induce the agent to accept a contract the principal guarantees that the contract generates the agent's reservation expected utility level:

$$(3.3.1) \quad EU \geq \bar{U}$$

The principal may face further incentive-compatibility constraints depending on the agent's ability to control the actions. If the agent effectively controls his own action (for instance, because it is unobservable to the principal), then his action satisfies the usual condition for expected utility maximization:

$$(3.3.2) \quad \alpha \in \arg\max_{\alpha'} EU(c_1, c_2, \alpha', \beta)$$

where the arguments of EU have been written explicitly.

Similarly, if the agent controls the principal's action, then the latter must satisfy:

$$(3.3.3) \quad \beta \in \arg\max_{\beta'} EU(c_1, c_2, \alpha, \beta')$$

The problems described in section 3.2 can now be stated formally. Since the agent's expected utility is guaranteed, the first-best situation is attained when the principal can effectively control both actions, so that the terms of compensation as well as the actions can be written into a contract. This case is described in (3.2.1). The optimum, denoted $(\hat{c}_1, \hat{c}_2, \hat{\alpha}, \hat{\beta})$, is the solution to

$$(3.3.4) \quad \text{Max}_{c_1, c_2, \alpha, \beta} EV \quad \text{s.t.} \quad (3.3.1)$$

The example in (3.2.2) allows the principal to write a contract specifying the terms of compensation and his own action, but not the agent's action. The agent then chooses an action satisfying (3.3.2). The optimum $(\bar{c}_1^*, \bar{c}_2^*, \bar{\alpha}^*, \bar{\beta}^*)$ is therefore the solution to

$$(3.3.5)^2 \quad \text{Max}_{c_1, c_2, \alpha, \beta} EV \quad \text{s.t.} \quad (3.3.1) \text{ and } (3.3.2)$$

In this form the model bears resemblance to the double moral hazard models discussed in section 3.1. However, whereas (3.3.5) allows the principal to take into account the agent's

reactions to changes in β (via (3.3.2)), Carmichael, and Cooper and Ross assume that all actions are chosen after the payments have been specified (in the second stage of the game), taking as given the actions (but not reactions) of the other(s). While (3.3.5) is like Emons' formulation, he assumes away the direct interdependence of the actions.

The nature of the relationship may be such that the agent controls the principal's action, but not his own. In this case, the principal chooses the compensation scheme and action of the agent, given the reactions of the agent, who chooses the principal's action after accepting a contract. Those reactions are captured by the incentive-compatibility constraint (3.3.3). The optimum $(\tilde{c}_1, \tilde{c}_2, \tilde{\alpha}, \tilde{\beta})$ is the solution to

$$(3.3.6) \quad \begin{array}{ll} \text{Max} & \text{EV} \\ c_1 \ c_2 \ \alpha \ \beta & \end{array} \quad \text{s.t.} \quad (3.3.1) \text{ and } (3.3.3)$$

The problems in (3.3.5) and (3.3.6) are second-best in the sense that, ex ante, the agent is indifferent among the situations $(\hat{c}_1, \hat{c}_2, \hat{\alpha}, \hat{\beta})$, $(\hat{c}_1^*, \hat{c}_2^*, \hat{\alpha}^*, \hat{\beta}^*)$ and $(\tilde{c}_1, \tilde{c}_2, \tilde{\alpha}, \tilde{\beta})$, while the principal generally prefers the first to either of the other two. Thus, the principal can face moral hazard because the agent's effort cannot be contracted upon, or because the agent is able to control the action of the principal.

A third-best situation allows the agent to choose both actions, exposing the principal to both kinds of moral hazard.

The principal then chooses the compensation scheme, again taking as given the reactions of the agent. This third-best optimum $(\overset{\circ}{c}_1, \overset{\circ}{c}_2, \overset{\circ}{\alpha}, \overset{\circ}{\beta})$ solves

$$(3.3.7) \quad \text{Max}_{c_1, c_2, \alpha, \beta} \quad EV \quad \text{s.t.} \quad (3.3.1) \quad (3.3.2) \quad \text{and} \quad (3.3.3)$$

The following sections characterize the solutions to problems (3.3.4)-(3.3.7).³

3.4 CHARACTERIZATION OF CONTRACTS WHEN THE PRINCIPAL CONTROLS BOTH ACTIONS (Problem (3.3.4))

Proposition 1

When the principal controls the compensation scheme and both actions, an interior optimum $(\hat{c}_1, \hat{c}_2, \hat{\alpha}, \hat{\beta})$ is characterized by:⁴

$$(3.4.1) \quad \frac{V'(w_1 - \hat{c}_1)}{V'(w_2 - \hat{c}_2)} = \frac{U'(\hat{c}_1)}{U'(\hat{c}_2)}$$

$$(3.4.2) \quad - \frac{\hat{\pi}^\alpha_{\Delta \hat{V}}}{\hat{\pi}^\beta_{\Delta \hat{V} - v}} = - \frac{\hat{\pi}^\alpha_{\Delta \hat{U} - u}}{\hat{\pi}^\beta_{\Delta \hat{U}}}$$

$$(3.4.3) \quad \frac{\hat{\pi}^\alpha_{\Delta \hat{V}}}{V'(w_i - \hat{c}_i)} = - \frac{\hat{\pi}^\alpha_{\Delta \hat{U} - u}}{U'(\hat{c}_i)} \quad i=1, 2$$

Proof: Let σ denote the multiplier associated with constraint (3.3.1). An interior solution satisfies:⁵

$$(3.4.4) \quad -V'(\hat{i}) + \hat{\sigma}U'(\hat{i}) = 0 \quad i=1,2$$

$$(3.4.5) \quad \hat{\pi}^\alpha \Delta \hat{V} + \hat{\sigma}(\hat{\pi}^\alpha \Delta \hat{U} - u) = 0$$

$$(3.4.6) \quad \hat{\pi}^\beta \Delta \hat{V} - v + \hat{\sigma} \hat{\pi}^\beta \Delta \hat{U} = 0$$

$$(3.4.7) \quad E\hat{U} - \bar{U} = 0$$

Proposition 1 follows from a rearrangement of (3.4.4)-(3.4.6). ■

The contract $(\hat{c}_1, \hat{c}_2, \hat{\alpha}, \hat{\beta})$ can be derived from (3.4.1)-(3.4.3) and (3.4.7). (3.4.2) and (3.4.3) imply

$$(3.4.8) \quad - \frac{\hat{\pi}^\beta \Delta \hat{V} - v}{V'(w_i - \hat{c}_i)} = \frac{\hat{\pi}^\beta \Delta \hat{U}}{U'(\hat{c}_i)} \quad i=1,2$$

Let the outcome be thought of as income. Then (3.4.1)-(3.4.3) and (3.4.8) state that the marginal rate of substitution between, respectively, income in the two states, the two actions, and either action and income in either state, are equal for the principal and agent. Condition (3.4.1) is well-known, and means that the principal and agent perfectly share the risks associated with the outcome (Borch, 1962).

3.5 CHARACTERIZATION OF CONTRACTS WHEN THE PRINCIPAL IS UNABLE TO CONTROL THE AGENT'S ACTION (Problem (3.3.5))

Proposition 2

When the principal controls the compensation scheme and his own action, and the agent's action is under the control of the agent, an interior optimum $(c_1^*, c_2^*, \alpha^*, \beta^*)$ is characterized by:

$$(3.5.1) \quad \frac{V'(w_1 - c_1^*)}{V'(w_2 - c_2^*)} < \frac{U'(c_1^*)}{U'(c_2^*)}$$

$$(3.5.2) \quad - \frac{\pi^{\alpha^*} \Delta V^*}{\pi^{\beta^*} \Delta V - V} > - \frac{\pi^{\alpha^*} \Delta U^* - u}{\pi^{\beta^*} \Delta U} = 0$$

$$(3.5.3) \quad - \frac{\pi^{\beta^*} \Delta V^* - V}{V'(w_1 - c_1^*)} > \frac{\pi^{\beta^*} \Delta U^*}{U'(c_1^*)}$$

$$(3.5.4) \quad - \frac{\pi^{\beta^*} \Delta V^* - V}{V'(w_2 - c_2^*)} > \frac{\pi^{\beta^*} \Delta U^*}{U'(c_2^*)} \quad \text{iff} \quad \frac{\pi^{\alpha\beta}}{\pi^{\beta}} > \frac{\pi^{\alpha}}{\pi}$$

Proof: Given that π is strictly concave in α , and that $\underline{\alpha}$ is suboptimal,⁶ (3.3.2) may be replaced by the first-order condition with respect to α of EU:⁷

$$(3.5.5) \quad EU^{\alpha} = \pi^{\alpha} \Delta U - u = 0$$

Let μ denote the multiplier associated with (3.5.5). An interior solution to problem (3.3.5) must satisfy:⁵

$$(3.5.6) \quad -\pi_i^* V'(\bar{i}) + \sigma \pi_i^* U'(\bar{i}) + \mu \pi_i^* U'(\bar{i}) = 0$$

$$i=1,2 \quad \pi_i = \begin{cases} 1-\pi & \text{if } i=1 \\ \pi & \text{if } i=2 \end{cases}$$

$$(3.5.7) \quad \pi^\alpha \Delta \bar{V} + \sigma (\pi^\alpha \Delta \bar{U} - u) + \mu \pi^\alpha \Delta \bar{U} = 0$$

$$(3.5.8) \quad \pi^\beta \Delta \bar{V} - v + \sigma \pi^\beta \Delta \bar{U} + \mu \pi^\beta \Delta \bar{U} = 0$$

$$(3.5.9) \quad E\bar{U} - \bar{U} = 0$$

$$(3.5.10) \quad \pi^\alpha \Delta \bar{U} - u = 0$$

Substituting (3.5.10) into (3.5.7) and rearranging gives

$$(3.5.11) \quad \mu = - \frac{\pi^\alpha \Delta \bar{V}}{\pi^\alpha \Delta \bar{U}}$$

Substituting (3.5.11) into (3.5.8) and rearranging gives

$$(3.5.12) \quad \sigma = - \frac{\pi^\beta \Delta \bar{V} - v - \pi^\alpha [\pi^{\alpha\beta} / \pi^{\alpha\alpha}] \Delta \bar{V}}{\pi^\beta \Delta \bar{U}}$$

Substituting (3.5.11) and (3.5.12) into (3.5.6) and rearranging gives

$$(3.5.13) \quad \frac{V'(\bar{i})}{U'(\bar{i})} = - \frac{\pi^\beta \Delta \bar{V} - v - \pi^\alpha [\pi^{\alpha\beta} / \pi^{\alpha\alpha}] \Delta \bar{V}}{\pi^\beta \Delta \bar{U}} - \frac{\pi_i^\alpha \pi^\alpha \Delta \bar{V}}{\pi_i^\alpha \pi^\alpha \Delta \bar{U}} \quad i=1,2$$

c_1^* , c_2^* , α^* and β^* are determined by (3.5.9), (3.5.10) and (3.5.13).

It is well-known that (3.5.13) implies (3.5.1), and ΔU^* , $\Delta V^* > 0$ (see, for instance, Grossman and Hart, 1983).

Hence, $\pi^{\beta} \Delta V^* - v < 0$ (using (3.5.8)) and $\pi^{\alpha} \Delta V^* > 0$. This with (3.5.10) gives (3.5.2).

Noting that $\pi_1 = 1 - \pi$ and $\pi_1^{\alpha} = -\pi^{\alpha}$, (3.5.13) with $i=1$ gives

$$-\frac{\pi^{\beta} \Delta V^* - v}{\pi^{\beta} \Delta U^*} > \frac{V'(\bar{1})}{U'(\bar{1})}, \text{ and hence (3.5.3).}$$

(3.5.13) with $i=2$ gives (3.5.4). ■

Condition (3.5.1) is a familiar result and implies that $c_1^* < c_2^*$. That is, the principal gives a reward when the outcome is high in order to motivate the agent.

Since the agent's effort at the margin is of no value to the agent himself (his private marginal cost and benefit from the action are equal), the principal's marginal rate of substituting the agent's action for his own exceeds the agent's marginal rate of substitution between the actions. This is condition (3.5.2).

The model contains a new incentive mechanism, namely, an increase in the principal's action, which raises the agent's marginal product (π^{α}) thus motivating higher effort by the agent (as well as directly increasing π). This incentive

effect is reflected in the term $\pi^{\alpha\beta}$ in (3.5.13). Since the principal takes into account the incentive effect of his own action, and the disincentive effect of a higher payment in state 1, while the agent does not, the principal places a higher value (at the margin) on his own action in terms of income in state 1, than does the agent. This is stated in (3.5.3).

The same need not hold when the valuation is in terms of income in state 2, because higher payments in state 2 motivate the agent. However, if the incentive effect of the principal's effort is stronger than the incentive effect of the state 2 payment (more precisely, if $\pi^{\alpha\beta}/\pi^{\beta} > \pi^{\alpha}/\pi$),⁸ then in terms of income in state 2, the principal values his own action more highly than does the agent. The converse also holds. This explains (3.5.4).

3.6 CHARACTERIZATION OF CONTRACTS WHEN THE PRINCIPAL IS UNABLE TO CONTROL HIS OWN ACTION (Problem (3.3.6))

In problems (3.3.4) and (3.3.5) an interior solution for all variables is a plausible assumption. Specifically, an increase in c_1 , c_2 , α or β imposes a cost upon the principal (an increase in α imposes an indirect cost, since the agent must be compensated by an increase in the expected payment).

It therefore seems reasonable that the optimum does not set these variables at their upper bounds. Likewise, if $\alpha=0$ or $\beta=0$, then $\pi=0$, so that the worst outcome is certain. Hence, α and β will not be set at their lower bounds. Further, since the agent is risk-averse, $c_1=0$ or $c_2=0$ is not optimal.

In contrast, in problem (3.3.6) it is very likely that either $\bar{\beta}$ or $\underline{\beta}$ will be chosen. Since an increase in β benefits, but does not involve a cost to the agent, a higher payment in state 2 induces the agent to dictate the maximum effort $\bar{\beta}$ to the principal. Conversely, a lower payment in state 2 induces the agent to choose $\underline{\beta}$. It is assumed that the principal wishes to avoid $\underline{\beta}$, and must therefore set the terms of compensation so that $\Delta U \geq 0$.

In view of these comments and the strict concavity of π in β , the constraint in (3.3.3) may be replaced by

$$(3.6.1)^9 \quad \pi^\beta \Delta U \geq 0 \quad \text{if } > \text{ then } \beta = \bar{\beta} \\ \text{if } = \text{ then } \beta \in (\underline{\beta}, \bar{\beta})$$

In addition to assuming away the possibility that $\underline{\beta}$ will be chosen, it is assumed that, given any contract which meets the agent's reservation expected utility, a choice of $\bar{\beta}$ by the agent is not the principal's most preferred choice; that is, given payments and actions satisfying the agent's participation constraint $EU(c_1, c_2, \alpha, \bar{\beta}) = \bar{U}$, the principal prefers a lower value of β . This seems a reasonable assumption, because if $\bar{\beta}$ were

optimal, then the principal and agent would not be in conflict over the value of β , so that the problem would not be of much interest. This is expressed formally in (3.6.10) below.

In choosing the optimal contract, the principal may choose to bear all of the risk ($c_1=c_2$), in which case the agent (but not the principal) is indifferent to the value of β , and is willing to choose the value that is optimal for the principal. Alternatively, the principal shifts some of the risk to the agent ($c_1 < c_2$), but thereby induces the agent to specify the costliest action $\bar{\beta}$. The outcome depends on the nature of the principal's utility function. The results below are confined to the class of utility functions V , which have a constant coefficient of absolute risk aversion; that is, $r = -V''(\cdot)/V'(\cdot)$ is a constant.

Lemma 1 identifies a utility function for the principal, such that the (constrained) optimum is characterized by the costliest action $\bar{\beta}$:

Lemma 1

Let the principal control the compensation scheme and the agent's action, and let the agent control the principal's action. Let the optimum be denoted $(\tilde{c}_1, \tilde{c}_2, \tilde{\alpha}, \tilde{\beta})$. Then there exists $r_1 > 0$ and $v_1 > 0$, such that $\tilde{\beta} = \bar{\beta}$.

Proof: Given $r_1 > 0$, let $v = 0$. Then $\tilde{\beta} = \bar{\beta}$. By continuity of v , given $r = r_1$, $\exists v_1 > 0$, such that $\tilde{\beta} = \bar{\beta}$. ■

Although, the principal may be better off if $\beta < \bar{\beta}$ (other things equal), the cost of inducing the agent to choose $\beta < \bar{\beta}$ is too high when $r = r_1$ and $v = v_1$ (that is, the cost of bearing all of the risk is high relative to the cost of implementing $\bar{\beta}$).

Proposition 3 describes the solution to (3.3.6) for a risk-neutral principal, and for a sufficiently risk-averse principal, given that the principal, other things equal, prefers $\beta < \bar{\beta}$ to $\beta = \bar{\beta}$:

Proposition 3

Let the principal control the compensation scheme and the agent's action, and let the agent control the principal's action. Assume that the optimum is interior for c_1 , c_2 , and α , and that $\underline{\beta}$ is never optimal. Further, if \tilde{L} is the Lagrangean of (3.3.6), assume that $\tilde{L}^\beta(c_1, c_2, \alpha, \bar{\beta}) < 0 \forall (c_1, c_2, \alpha, \bar{\beta})$ satisfying (3.3.1).

(a) If $r = 0$ (the principal is risk-neutral), then $(\tilde{c}_1, \tilde{c}_2, \tilde{\alpha}, \tilde{\beta})$ is characterized by

$$(3.6.2) \quad \tilde{c}_1 = \tilde{c}_2$$

$$(3.6.3) \quad - \frac{\tilde{\pi}^\beta \Delta \tilde{V} - v}{\tilde{\pi}^\alpha \Delta \tilde{V}} = - \frac{\tilde{\pi}^\beta \Delta \tilde{U}}{\tilde{\pi}^\alpha \Delta \tilde{U} - u} = 0$$

$$(3.6.4) \quad \frac{\tilde{\pi}^\alpha \Delta \tilde{V}}{k} = \frac{u}{U'(\tilde{c}_i)} \quad i=1,2 \quad \text{where } k=V'(\cdot)$$

(b) Let $r=r_1$ and $v=v_1$ (as defined in Lemma 1). Then

$(\tilde{c}_1, \tilde{c}_2, \tilde{\alpha}, \tilde{\beta})$ is characterized by

$$(3.6.5) \quad \frac{V'(w_1 - \tilde{c}_1)}{V'(w_2 - \tilde{c}_2)} = \frac{U'(\tilde{c}_1)}{U'(\tilde{c}_2)}$$

$$(3.6.6) \quad - \frac{\tilde{\pi}^\beta \Delta \tilde{V} - v}{\tilde{\pi}^\alpha \Delta \tilde{V}} > - \frac{\tilde{\pi}^\beta \Delta \tilde{U}}{\tilde{\pi}^\alpha \Delta \tilde{U} - u}$$

$$(3.6.7) \quad \frac{\tilde{\pi}^\alpha \Delta \tilde{V}}{V'(w_i - \tilde{c}_i)} = - \frac{\tilde{\pi}^\alpha \Delta \tilde{U} - u}{U'(\tilde{c}_i)} \quad i=1,2$$

Proof: Let γ denote the multiplier associated with (3.6.1). A solution to problem (3.3.6), replacing (3.3.3) with (3.6.1) satisfies⁵

$$(3.6.8) \quad -\tilde{\pi} \, y'(\tilde{I}) + \tilde{\sigma} \tilde{\pi} \, \psi'(\tilde{I}) + \tilde{\gamma} \tilde{\pi}^\beta \psi'(\tilde{I}) = 0 \quad i=1,2$$

$$(3.6.9) \quad \tilde{\pi}^\alpha \Delta \tilde{V} + \tilde{\sigma} (\tilde{\pi}^\alpha \Delta \tilde{U} - u) + \tilde{\gamma} \tilde{\pi}^\beta \Delta \tilde{U} = 0$$

$$(3.6.10) \quad \tilde{\pi}^{\beta} \Delta \tilde{V} - v + \tilde{\sigma} \tilde{\pi}^{\beta} \Delta \tilde{U} + \tilde{\gamma} \tilde{\pi}^{\beta \beta} \Delta \tilde{U} \leq 0 \quad < \text{ if } \beta = \bar{\beta}$$

(by the assumption in the proposition)

$$(3.6.11) \quad E\tilde{U} - \bar{U} = 0$$

$$(3.6.12) \quad \tilde{\pi}^{\beta} \Delta \tilde{U} \geq 0 \quad \text{if } > \text{ then } \beta = \bar{\beta} \text{ and } \tilde{\gamma} = 0$$

(a) Let $r=0$. Let $\tilde{c}_i = \hat{c}_i$ $i=1,2$, and $\tilde{\alpha} = \hat{\alpha}$ (where \hat{c}_1 , \hat{c}_2 , and $\hat{\alpha}$ are part of the solution to problem (3.3.4) in which the principal controls all the variables).

It is well-known that if V' is constant (3.4.1) implies $\hat{c}_1 = \hat{c}_2$, so that $\Delta \tilde{U} = 0$. Hence, $\pi^{\beta} \Delta \tilde{U} = 0 \forall \beta$. In particular, $\hat{\beta}$ satisfies $\pi^{\beta} \Delta \tilde{U} = 0$, so that, given $(\hat{c}_1, \hat{c}_2, \hat{\alpha})$, $\hat{\beta}$ is incentive-compatible. Thus, when $r=0$, the solution to (3.3.6) is the first-best solution. Proposition 3a then follows from Proposition 1 with $\hat{\Delta \tilde{U}} = 0$ and $V'(\cdot) = k$.

(b) By Lemma 1, $\tilde{\beta} = \bar{\beta}$. If $\Delta \tilde{U} \leq 0$, then the agent chooses $\beta < \bar{\beta}$, which gives a contradiction. Hence, $\Delta \tilde{U} > 0$, so that $\tilde{\gamma} = 0$ by (3.6.12). Substituting $\tilde{\gamma}$ into (3.6.8), (3.6.9) and (3.6.10) (for which the inequality applies) and rearranging gives (3.6.5)-(3.6.7). ■

If the principal is risk-neutral, the optimal payments and actions are determined by (3.6.2), (3.6.3), (3.6.4) and

(3.6.11). The first-best outcome can be achieved by shielding the agent from all risk. A fixed payment given by (3.6.2) makes the agent indifferent to the principal's action, so that the principal can effectively choose his own action (that is, having specified \hat{c}_1 , \hat{c}_2 and $\hat{\alpha}$ in the contract, $\hat{\beta}$ is incentive-compatible).

At an optimum neither party is affected by marginal changes in the principal's action, the agent being indifferent to that action, and the principal having equated the marginal benefit ($\pi^\beta \Delta V$) with the marginal cost (v) of his action. Thus, the marginal rates of substituting the agent's for the principal's action is zero for both principal and agent (condition (3.6.3)).

The result suggests (although this is not proven), that when the gains from risk-sharing are small relative to the cost of $\bar{\beta}$ (r is relatively small), it is optimal to sacrifice all risk-sharing opportunities to avoid moral hazard (a choice of $\bar{\beta}$) by the agent.

When the gains from risk-sharing are large relative to the cost of $\bar{\beta}$, the optimal payments and actions are determined by (3.6.5), (3.6.7), (3.6.11) and $\tilde{\beta} = \bar{\beta}$. In this case, the principal shifts some of the risk to the agent. But, as soon as the agent bears any risk, he dictates the costliest action to the principal. Given that the principal controls the agent's action, the compensation scheme generates no

(dis)incentives other than motivating $\bar{\beta}$ when $c_1 < c_2$. Thus, having decided to share some risk it is optimal to share risk perfectly.

It seems possible that, if the principal wants to share risks, and hence accepts $\bar{\beta}$, he will require a greater than first-best effort from the agent. Specifically, if the principal takes as given $\beta = \bar{\beta}$, then the problem amounts to (3.3.4) with $\beta = \bar{\beta}$ replacing (3.4.6). Suppose, initially, that the principal controls β , and hence specifies the first-best contract. Then (3.4.4)-(3.4.7) are satisfied. Now replace $\hat{\beta}$ with $\bar{\beta}$. If \hat{L} is the Lagrangean of (3.3.4), then $\hat{L}^{c_1}(\hat{c}_1, \hat{\alpha}, \bar{\beta}, \hat{\sigma}) = 0$ from (3.4.4) (no direct effect on the compensation scheme), $\hat{L}^{\alpha}(\hat{c}_1, \hat{\alpha}, \bar{\beta}, \hat{\sigma}) > 0$ from (3.4.5), and $\hat{L}^{\sigma}(\hat{c}_1, \hat{\alpha}, \bar{\beta}, \hat{\sigma}) > 0$ from (3.4.7). This suggests that the principal, when losing control over his own action, adjusts the agent's action upwards, meeting the agent's reservation expected utility by adjusting the payments.

3.7 CHARACTERIZATION OF CONTRACTS WHEN THE AGENT CONTROLS BOTH ACTIONS (Problem (3.3.7))

If the principal offers a fixed payment (to make all values of β incentive-compatible), then the agent will choose $\alpha = \underline{\alpha}$, and so guarantee the unfavorable outcome. To avoid this

possibility the agent must be paid more in the favorable state. This induces the agent to dictate $\bar{\beta}$ to the principal. The characterization of the optimal compensation then follows directly from existing results, since the problem reduces to the model in (3.3.5); that is, there is imperfect risk-sharing to motivate a higher effort on the part of the agent. Further, at the optimum, the actions do not satisfy the efficiency condition (3.4.2). These observations are summarized in

Proposition 4

Let the principal control the compensation scheme, and let the agent control both actions. Assume that the optimum $(\dot{c}_1, \dot{c}_2, \dot{\alpha}, \dot{\beta})$ is interior for c_1, c_2 and α and that $\underline{\beta}$ is never optimal. Further, if \dot{L} is the Lagrangean of (3.3.7), assume that $\dot{L}^{\dot{\beta}} < 0 \forall (c_1, c_2, \alpha, \bar{\beta})$ satisfying (3.3.1). Then $(\dot{c}_1, \dot{c}_2, \dot{\alpha}, \dot{\beta})$ is characterized by

$$(3.7.1) \quad \frac{V'(w_1 - \dot{c}_1)}{V'(w_2 - \dot{c}_2)} < \frac{U'(\dot{c}_1)}{U'(\dot{c}_2)}$$

$$(3.7.2) \quad - \frac{\pi^{\dot{\alpha}} \Delta \dot{V}}{\pi^{\dot{\beta}} \Delta \dot{V} - v} > - \frac{\pi^{\dot{\alpha}} \Delta \dot{U} - u}{\pi^{\dot{\beta}} \Delta \dot{U}} = 0$$

$$(3.7.3) \quad - \frac{\pi^{\dot{\beta}} \Delta \dot{V} - v}{V'(w_1 - \dot{c}_1)} > \frac{\pi^{\dot{\beta}} \Delta \dot{U}}{U'(\dot{c}_1)}$$

$$(3.7.4) \quad - \frac{\overset{\circ}{\pi}^{\beta} \Delta \overset{\circ}{V} - v}{V'(\overset{\circ}{w}_2 - \overset{\circ}{c}_2)} > \frac{\overset{\circ}{\pi}^{\beta} \Delta \overset{\circ}{U}}{U'(\overset{\circ}{c}_2)} \quad \text{if, given } \beta = \bar{\beta} \quad \frac{\pi^{\alpha\beta}}{\pi^{\alpha}} \geq \frac{\pi^{\alpha}}{\pi}$$

Proof: A solution $(\overset{\circ}{c}_1, \overset{\circ}{c}_2, \overset{\circ}{\alpha}, \overset{\circ}{\beta})$ to (3.3.7) (replacing (3.3.3) with (3.6.1)) satisfies⁵

$$(3.7.5) \quad -\overset{\circ}{\pi}_i V'(\overset{\circ}{i}) + \overset{\circ}{\sigma} \overset{\circ}{\pi}_i U'(\overset{\circ}{i}) + \overset{\circ}{\mu} \overset{\circ}{\pi}_i^{\alpha} U'(\overset{\circ}{i}) + \overset{\circ}{\gamma} \overset{\circ}{\pi}_i^{\beta} U'(\overset{\circ}{i}) = 0 \quad i=1,2$$

$$(3.7.6) \quad \overset{\circ}{\pi}^{\alpha} \Delta \overset{\circ}{V} + \overset{\circ}{\sigma} (\overset{\circ}{\pi}^{\alpha} \Delta \overset{\circ}{U} - u) + \overset{\circ}{\mu} \overset{\circ}{\pi}^{\alpha\alpha} \Delta \overset{\circ}{U} + \overset{\circ}{\gamma} \overset{\circ}{\pi}^{\alpha\beta} \Delta \overset{\circ}{U} = 0$$

$$(3.7.7) \quad \overset{\circ}{\pi}^{\beta} \Delta \overset{\circ}{V} - v + \overset{\circ}{\sigma} \overset{\circ}{\pi}^{\beta} \Delta \overset{\circ}{U} + \overset{\circ}{\mu} \overset{\circ}{\pi}^{\alpha\beta} \Delta \overset{\circ}{U} + \overset{\circ}{\gamma} \overset{\circ}{\pi}^{\beta\beta} \Delta \overset{\circ}{U} \leq 0 \quad < \text{ if } \beta = \bar{\beta}$$

(by the assumption in the proposition).

$$(3.7.8) \quad E\overset{\circ}{U} - \bar{U} = 0$$

$$(3.7.9) \quad \overset{\circ}{\pi}^{\alpha} \Delta \overset{\circ}{U} - u = 0$$

$$(3.7.10) \quad \overset{\circ}{\pi}^{\beta} \Delta \overset{\circ}{U} \geq 0 \quad \text{if } > \text{ then } \overset{\circ}{\beta} = \bar{\beta} \text{ and } \overset{\circ}{\gamma} = 0$$

To avoid $\alpha = \underline{\alpha}$ the principal sets $\Delta \overset{\circ}{U} > 0$, so that $\overset{\circ}{\beta} = \bar{\beta}$ and $\overset{\circ}{\gamma} = 0$.

Substituting $\overset{\circ}{\gamma}$ and (3.7.9) into (3.7.6) gives

$$(3.7.11) \quad \overset{\circ}{\mu} = - \frac{\overset{\circ}{\pi}^{\alpha} \Delta \overset{\circ}{V}}{\overset{\circ}{\pi}^{\alpha\alpha} \Delta \overset{\circ}{U}}$$

Substituting $\overset{\circ}{\gamma}$ and (3.7.11) into (3.7.5) gives

$$(3.7.12) \quad \frac{V'(\dot{2})}{U'(\dot{2})} - \frac{V'(\dot{1})}{U'(\dot{1})} = - \frac{(\dot{\pi}^\alpha)^2 \Delta \dot{V}}{(1-\dot{\pi}) \dot{\pi} \dot{\pi}^{\alpha\alpha} \Delta \dot{U}}$$

\dot{c}_1 , \dot{c}_2 , $\dot{\alpha}$ and $\dot{\beta}$ are determined by (3.7.8), (3.7.9), (3.7.12) and $\dot{\beta} = \bar{\beta}$.

(3.7.12) implies (3.7.1). Hence $\Delta \dot{V}$, $\Delta \dot{U} > 0$, so that with $\dot{\gamma} = 0$, (3.7.7) implies $\dot{\pi}^\beta \Delta \dot{V} - v < 0$. This with (3.7.9) gives (3.7.2). Substituting $\dot{\gamma}$ and (3.7.11) into (3.7.7) gives

$$(3.7.13) \quad \dot{\sigma} < - \frac{\dot{\pi}^\beta \Delta \dot{V} - v - \dot{\pi}^\alpha [\dot{\pi}^{\alpha\beta} / \dot{\pi}^{\alpha\alpha}] \Delta \dot{V}}{\dot{\pi}^\beta \Delta \dot{U}}$$

Substituting (3.7.11) and (3.7.13) into (3.7.5) and rearranging gives

$$(3.7.14) \quad \frac{V'(\dot{i})}{U'(\dot{i})} < - \frac{\dot{\pi}^\beta \Delta \dot{V} - v - \dot{\pi}^\alpha [\dot{\pi}^{\alpha\beta} / \dot{\pi}^{\alpha\alpha}] \Delta \dot{V}}{\dot{\pi}^\beta \Delta \dot{U}} - \frac{\dot{\pi}_1^\alpha \dot{\pi}^\alpha \Delta \dot{V}}{\dot{\pi}_1 \dot{\pi}^{\alpha\alpha} \Delta \dot{U}} \quad i=1,2$$

$$(3.7.14) \text{ with } i=1 \text{ gives } - \frac{\dot{\pi}^\beta \Delta \dot{V} - v}{\dot{\pi}^\beta \Delta \dot{U}} > \frac{V'(\dot{1})}{U'(\dot{1})}, \text{ and hence (3.7.3).}$$

$$(3.7.14) \text{ with } i=2 \text{ gives (3.7.4).} \quad \blacksquare$$

For reasons discussed in section 3.5 (which differs from this section in that here the agent controls β), at the optimum there is imperfect risk-sharing, and the agent's marginal rate of substituting his own for the principal's action is lower

than that for the principal.

The principal's effort is higher than optimal, being chosen by the agent who does not consider its cost. This is reflected in the discrepancies between the marginal rates of substituting effort for income in (3.7.3) and (3.7.4). However, some of the discrepancy is desired by the principal, because higher effort by himself motivates the agent. This is the incentive effect discussed in section 3.5.

It seems possible that the agent will choose higher effort for himself when he also controls the principal's action, than otherwise (that is, $\alpha^{\circ} > \alpha^*$ may hold). If $\beta = \bar{\beta}$, the principal's problem reduces to problem (3.3.5), replacing (3.5.8) with $\beta = \bar{\beta}$. The higher effort by the principal makes the agent's effort more productive at the margin. This induces the agent to choose higher effort. It also motivates the principal, who captures some of the increase in the agent's productivity, to provide a further inducement by shifting more risk to the agent.

Consider, for instance, a risk-neutral principal with $V'(\cdot) \equiv k$. Rearranging (3.5.6) and using (3.5.11) gives

$$(3.7.15) \quad \frac{k}{U'(\frac{*}{2})} - \frac{k}{U'(\frac{*}{1})} = - \frac{(\pi^{\alpha})^2 \Delta \bar{V}}{(1-\pi)^* \pi^* \pi^{\alpha} \Delta \bar{U}}$$

If $\pi \geq \frac{1}{2}$, then replacing $\bar{\beta}$ with $\bar{\beta}$ causes π^{α} to increase and $(1-\pi)\pi$ to fall, in which case the increase in β directly raises

the right-hand side of (3.7.15). To maintain the equality, $c_2 - c_1$ must increase, motivating an increase in α . While this argument ignores the feedback effects from changes in the other first-order conditions, it suggests that there are forces which make it optimal for the principal to shift more risk to the agent, and for the agent to choose a closer to first-best effort.

3.8 CONCLUSION

A small literature treats the principal as an active rather than passive player in the principal-agent relationship. It observes that if the principal's actions are productive, then the principal as well as the agent may be a source of moral hazard, requiring a way of simultaneously attacking moral hazard on the part of both players.

The analysis above looks at a different kind of double moral hazard, namely on the part of the agent, who may himself shirk, or may be in a position to impose undesirable actions upon the principal. Under various assumptions about the ability of the players to control actions (but always allowing the principal to control the compensation scheme), optimal contracts are described in terms of their risk-sharing properties and the nature of the actions.

When there is no moral hazard the payments and actions

are chosen, so that at an optimum, the marginal rates of substitution between income across states, actions, or between actions and income in either state, respectively, are equal for the principal and agent.

When the agent is able to shirk, but cannot control the principal's action, some exposure to risk by the agent is optimal (for the well-known reason that it motivates a higher effort by the agent). Further, the principal can motivate the agent (as well as substitute for the agent's action) by adjusting his own action.

If the agent controls the principal's, but not his own action, then a risk-neutral principal makes a fixed payment to the agent, and, because the agent is indifferent to both actions, stipulates a first-best mix of actions. A sufficiently risk-averse principal chooses perfect risk-sharing, carries out the maximum effort, and may stipulate a higher than first-best effort for the agent.

If the agent controls both actions, the contract involves imperfect risk-sharing, maximum effort by the principal, while the agent's effort may be closer to being first-best than the level he chooses when the principal's action is not under his control.

3.9 NOTES

1. For instance, workers (by way of unions) might be able to enforce excessive levels of safety in firms; lawyers may advise clients to carry out actions, which given greater knowledge of the law, the client would not have carried out; and conditions in warranties may lead the buyer to take more care than if he had fully understood the nature of product. In addition, the worker and the lawyer have an incentive to shirk, and the seller has an incentive to produce a good of low quality.

2. See section 1.2 and note 8 in Chapter 1 for a discussion of α as a variable under the maximization sign, when the agent controls the value of α .

3. Problems (3.3.6) and (3.3.7) are considered in chapter 5 in a similar context. In this model the graduation rate depends on student effort α (controlled by the government or by students), and the level of education β (controlled by students).

4. Notation is as follows:

$\hat{f} = f(\hat{\cdot})$ (likewise for symbols other than \wedge)

$V(i) = V(w_i - c_i)$ $U(i) = U(c_i)$ $\Delta V = V(2) - V(1)$ $\Delta U = U(2) - U(1)$

f' is the derivative of f

f^x is the partial derivative of f with respect to x

5. It is assumed that the Hessian matrix of this problem is

negative semi-definite, so that the conditions are also sufficient for a (local) maximum.

6. If $\underline{\alpha}$ is optimal, then $\pi(\underline{\alpha}, \beta) = 0$ is optimal, and the principal would not hire the agent.

7. If $\Delta U \leq 0$, then $\alpha = \underline{\alpha}$. Hence $\Delta U > 0$. The second-order condition for maximum expected utility of the agent - namely $\pi^{*\alpha\alpha} \Delta U^* < 0$ - is therefore satisfied, and α^* maximizes $EU(c_1^*, c_2^*, \alpha^*, \beta^*)$. This is a special case of the first-order approach described in Rogerson (1985).

8. The effect of an increase in β is to raise π^α , and induce higher effort from the agent. $\pi^{\alpha\beta}/\pi^\beta$ is a measure of the incentive effect of β .

The effect of an increase in c_2 is to raise $EU^\alpha = \pi^\alpha (U(c_2) - U(c_1))$, which induces higher effort by the agent. π^α/π is a measure of the incentive effect of c_2 .

9. The second-order condition for an expected utility maximum of the agent is $\pi^{\beta\beta} \Delta U < 0$.

It is argued below that if $\Delta U = 0$, any value of β maximizes the agent's expected utility. If $\Delta U > 0$, then the second-order condition is satisfied, so that β again maximizes the agent's expected utility.

CHAPTER 4

THE CORE OF AN ECONOMY WITH MORAL HAZARD

4.1 INTRODUCTION

Edgeworth (1881), and Debreu and Scarf (1963) identified the allocations that survive the bargaining process in a private goods economy. Since then Shapley and Shubik (1969), Foley (1970), Richter (1974), Wooders (1978, 1981), and Telser (1982), among others, have extended the theory to economies with public goods. They observe that, in the presence of public goods, coalitions of agents may not be able to make themselves completely independent from the rest of society. A theory of the core, therefore, requires a statement about the reaction of noncoalition agents to the formation of a coalition.

A question addressed below is whether the result, that (under certain conditions) the core of a sufficiently large private goods economy approximates a competitive allocation, also holds for an economy with public goods. The existing literature does not appear to resolve the issue. It is investigated below in the context of a moral hazard model. In this model agents receive a state-dependent endowment of a private good. There is a common probability distribution over

the states, which depends on actions by the agents. Better actions shift the distribution towards more favorable states. Since the action of any agent enters the expected utility of all agents, and since no one can be prevented from its benefits, each action has the properties of a public good. The problem is to determine which allocations of the private good and which actions are in the core, whether a core exists, and how the core is affected as the economy grows.

The model may be seen as a generalization of the principal-agent models of Ross (1973), Holmström (1979, 1982) and others. The possibility of moral hazard arises if coalition agents choose actions without regard to noncoalition agents, so that their actions are likely to be socially suboptimal. The model can be derived from standard principal-agent models by allowing the principal, as well as the agent, to choose an action, and by allowing the agent to take part in determining his own compensation.¹

The model may also be seen as a variation of a class of models developed by, among others, Hurwicz (1972), Townsend (1979), Postlewaite and Schmeidler (1986), and Palfrey and Srivastava (1986, 1989). These authors extend the Arrow-Debreu model of general equilibrium to situations where agents must be induced to reveal truthful information about a private characteristic such as preferences or endowments. In the generalized moral hazard model agents must be induced to choose

desirable actions. However, this variation creates a difference that is fundamental in an analysis of the core. In the models cited above, the welfare of a coalition of agents does not depend on the messages sent by noncoalition agents, because messages are of relevance only if commodity exchange takes place between senders and recipients. In contrast, in the moral hazard model, a coalition's welfare depends on actions by noncoalition agents, even if there is no commodity exchange between these groups.

The analysis is organized as follows. Section 4.2 gives an example. The problem is formalized in section 4.3, and diagrammatically illustrated in section 4.4 for an economy with two agents. Section 4.5 discusses Pareto optimality, and a definition of the core is given in section 4.6. The Pareto optimality of contracts in the core is established in section 4.7, while section 4.8 discusses individually rational contracts, and characterizes the core of an economy with one pair of agents. Section 4.9 extends the characterization to economies with r pairs of agents. Existence of the core is proven in section 4.10. Section 4.11 concludes.

4.2 AN EXAMPLE

Consider a group of farmers, each with his own orchard. In each orchard the apple yield is either high or low,

depending on whether or not growing conditions turn out to be favorable. One of the factors influencing growing conditions, and hence, the expected yield, is the incidence of disease. If disease breaks out in any orchard, it spreads to all other orchards, thereby lowering the probability that the yield will be high. To avoid disease all farmers must spray their orchards. Each farmer prefers that his neighbors spray, but because spraying is costly, he has an incentive to not spray his own trees.

The farmers write a contract stipulating an exchange of apples across states of nature in order to spread the risk associated with production, and specifying whether or not spraying is required. The incentive to sign a contract which requires spraying depends upon the associated risk-sharing arrangement. Suppose, for instance, that there are two farmers, one of whom agrees to give up apples if a low yield is realized, in exchange for apples if a high yield is realized. Exchange makes the low yield state more costly to that farmer, so that he has a greater incentive to spray his orchard. In contrast, the other farmer has less incentive to spray his orchard.

Under these circumstances what conditions (regarding preferences, technology, behavior, etc.) guarantee that the farmers collectively will agree on a contract which spreads the risks associated with the yield and requires every farmer to

spray his orchard? Further, what are the properties of such an agreement - is it efficient; does it discriminate among identical farmers, does it bear any relation to contracts traded in a competitive market?

In the above example a key feature is that disease can be avoided only if each orchard is sprayed. This feature also applies to problems such as the building of a dike, when more than one team of workers is involved. Shirking by one team, causing the dike to break at a single place, has the same effect as shirking by several teams, causing the dike to break at several places. Similarly, classified information held by a group of people can be leaked as effectively by a single member as by all members.

The following section describes the formal model.

4.3 THE MODEL

Consider the economy

$$\Gamma = \left[\{w^{hi}\}_{hi \in \kappa} , \{x_1^{hi}, x_2^{hi}\}_{hi \in \kappa} , \{A^{hi}\}_{hi \in \kappa} , \pi , \{V^{hi}\}_{hi \in \kappa} \right]$$

κ is a set of $2r$ agents, with r agents of each of two types. Agent hi is the i th agent of the h th type, and has a state-dependent endowment of a private good $w^{hi} = (w_1^{hi}, w_2^{hi})$, where w_j^{hi} is his endowment of the good if state j occurs. Without loss of generality it is assumed that state 2 is the

favorable state:

Assumption 1

$$w_1^{hi} < w_2^{hi} \quad \forall hi \in \kappa.$$

x_j^{hi} is the set of allocations x_j^{hi} of the good to agent hi if state j occurs. Let $x^{hi} = (x_1^{hi}, x_2^{hi})$ and let $x^{hi} = (x_1^{hi}, x_2^{hi})$ denote a member of x^{hi} (thus, x^{hi} is agent hi 's ex ante allocation of the good).

A^{hi} is the set of actions α^{hi} of agent hi , and is restricted to two possibilities: $A^{hi} = \{\underline{\alpha}, \bar{\alpha}\}$, $\underline{\alpha} < \bar{\alpha}$. Actions may be interpreted as levels of effort, and are assumed observable.

The probability that state 2 occurs² is a function of the effort of each agent, namely, $\pi: \prod_{\kappa} A^{hi} \rightarrow (0, 1)$, where $\prod_{\kappa} = \prod_{hi \in \kappa}$ denotes the product over all hi belonging to κ . It is assumed that the value of π is determined by the smallest of all effort levels, and that effort is productive in the sense that a greater minimum effort raises the probability of a large endowment:

Assumption 2

$$\begin{aligned} \pi(\alpha^{1l}, \dots, \alpha^{2r}) &= \pi(\min\{\alpha^{1l}, \dots, \alpha^{2r}\}) \\ &\in \{\pi(\underline{\alpha}), \pi(\bar{\alpha})\} \\ &= \{\underline{\pi}, \bar{\pi}\}, \quad \underline{\pi} < \bar{\pi}. \end{aligned}$$

v^{hi} is agent hi 's state-independent utility function over the good and agent hi 's action, and is assumed separable. Specifically, $v^{hi}: (x_j^{hi}, A^{hi}) \rightarrow R$ (where R is the set of non-negative real numbers), and $v^{hi}(x_j^{hi}, \alpha^{hi}) = U_j^{hi}(x_j^{hi}) - \alpha^{hi}$, where

Assumption 3

U^{hi} is differentiable, monotone increasing, and strictly concave $\forall hi \in \kappa$.

Notation involving subsets of agents is as follows. Θ denotes the set of non-empty subsets of κ , and a member of Θ (a coalition) is denoted θ . Let $S^\theta = \prod_\theta X^{hi} \times \prod_\kappa A^{hi}$ denote the set of strategies $s_\theta = (x_\theta, \alpha_\kappa)$, where x_θ denotes the vector of ex ante allocations for all members of θ , and α_κ denotes a vector of actions for all members of κ . Let $\underline{\alpha}_\kappa$ and $\bar{\alpha}_\kappa$ denote the 2r-vectors $(\underline{\alpha}, \dots, \underline{\alpha})$ and $(\bar{\alpha}, \dots, \bar{\alpha})$. $-\theta$ (the complementary coalition) is the complement of θ in κ .

For each $\hat{s}_\theta = (\hat{x}_\theta, \hat{\alpha}_\theta, \hat{\alpha}_{-\theta}) \in S^\theta$, Γ/\hat{s}_θ is the game induced on subgroup $-\theta$ by the choice \hat{s}_θ of coalition θ . In full

$$\Gamma/\hat{s}_\theta = \left[\{w^{hi}\}_{hi \in -\theta}, \{x_1^{hi}, x_2^{hi}\}_{hi \in -\theta}, \{A^{hi}\}_{hi \in -\theta}, \pi', \{v^{hi}\}_{hi \in -\theta} \right]$$

where $\pi': \prod_{-\theta} A^{hi} \rightarrow (0,1)$ is given by $\pi'(\alpha_{-\theta}) = \pi(\hat{\alpha}_\theta, \alpha_{-\theta})$.

Before the state occurs, agents have the following

information. Each agent knows the state-dependent endowment, the form of the probability function, and utility functions. Given this information, the agents sign a contract stipulating an allocation of the good and an action for each agent.³ The allocation is essentially a risk-sharing arrangement.

Agent h_i 's ex ante evaluation of an allocation x^{h_i} and a set of actions α_{κ} is

$$W^{h_i}(x^{h_i}, \alpha_{\kappa}) = (1-\pi)U^{h_i}(x_1^{h_i}) + \pi U^{h_i}(x_2^{h_i}) - \alpha^{h_i}$$

The problem is to identify the allocations and actions, which the group might collectively agree upon, and which might therefore be observed as written contracts.⁴

Like the models of Hurwicz, Townsend, etc., this model is an extension of the Arrow-Debreu commodity exchange model with uncertainty. Unlike those models, interaction among agents involves more than the exchange of a private good based on messages about private characteristics; even in the absence of exchange, agents interact because the action of each agent may affect the welfare of every other agent. More specifically, agent h_i 's action α^{h_i} enters the expected utility function of all agents, and no agent can be prevented from "consuming" the quantity α^{h_i} , so that this economy contains 2r public goods (the actions) as well as a private good.

4.4 DIAGRAMMATIC ILLUSTRATION

Consider an economy with one agent of each type. Agent $h1$, $h=1,2$, has convex indifference curves over the good in the two states, parameterized by the actions $(\alpha^{11}, \alpha^{21})$. Given a set of actions, higher indifference curves imply higher expected utility levels. A greater value of $\min\{\alpha^{11}, \alpha^{21}\}$ (and hence a greater value of π) raises the expected utility from the second state relative to the expected utility from the first state, and thus flattens the indifference curve through a given bundle \bar{x}^{h1} (see Figure 4.14.1 in section 4.14). Since there are four different pairs of actions, there are four indifference curves through \bar{x}^{h1} , but, since $\pi(\underline{\alpha}, \underline{\alpha}) = \pi(\underline{\alpha}, \bar{\alpha}) = \pi(\bar{\alpha}, \underline{\alpha})$ three of these indifference curves coincide with the steeper curve, differing only in the levels of expected utility associated with them.

Figure 4.14.2 shows an Edgeworth box with an endowment point w , and consumption of the good in each state, measured in the indicated directions. It shows the indifference curves of both agents through the endowment point. Without loss of generality, it is assumed that risk-sharing involves the type 2 agent giving up some of the good if state 1 occurs, in exchange for receiving more of the good if state 2 occurs; thus, $MRS^{11}(w^{11}, \alpha_{\kappa}^{11}) > MRS^{21}(w^{21}, \alpha_{\kappa}^{21})$, where $MRS^{hi}(x^{hi}, \alpha_{\kappa}^{hi})$ is agent hi 's marginal rate of substituting the good in state 1 for the good

in state 2, given the allocation x^{hi} and actions α_κ .

Allocations along the contract curve satisfy

$$\frac{\partial W^{11}/\partial x_1^{11}}{\partial W^{11}/\partial x_2^{11}} = \frac{\partial W^{21}/\partial x_1^{21}}{\partial W^{21}/\partial x_2^{21}} \quad \text{and thus} \quad \frac{\partial U^{11}/\partial x_1^{11}}{\partial U^{11}/\partial x_2^{11}} = \frac{\partial U^{21}/\partial x_1^{21}}{\partial U^{21}/\partial x_2^{21}},$$

which is independent of the actions.

4.5 PARETO OPTIMAL STRATEGIES

Let \hat{S}^κ denote the set of Pareto optimal strategies when actions are enforceable. Following the usual presentation of Pareto optima, each member $\hat{s}_\kappa = (\hat{x}_\kappa, \hat{\alpha}_\kappa) \in \hat{S}^\kappa$ is the solution to a problem of the form

$$(4.5.1) \quad \text{Max}_{x_\kappa, \alpha_\kappa} W^{11}(x^{11}, \alpha_\kappa)$$

$$(4.5.2) \quad \text{s.t.} \quad \sum_\kappa x_j^{hi} = \sum_\kappa w_j^{hi} \quad j=1,2$$

$$(4.5.3) \quad \text{s.t.} \quad W^{hi}(x^{hi}, \alpha_\kappa) \geq \bar{W}^{hi} \quad \forall hi \in \kappa \quad hi \neq 11$$

where $\sum_\kappa \equiv \sum_{hi \in \kappa}$ denotes the sum over all hi belonging to κ ; and \bar{W}^{hi} is a given level of expected utility for agent hi . To derive the other members of \hat{S}^κ , \bar{W}^{hi} is varied over all feasible values of W^{hi} , for all $hi \neq 11$.

Let \bar{S}^{κ} denote the set of Pareto optimal strategies when actions are unenforceable. Assuming that each agent chooses his own action under Nash conjectures, α_{κ} is an equilibrium vector of actions if

$$(4.5.4) \quad \alpha_{hi}^{hi} \in \operatorname{argmax}_{\alpha_{hi}^{hi}} W^{hi}(x^{hi}, \alpha_{hi}^{hi}, \alpha_{\kappa \setminus \{hi\}}) \quad \forall hi \in \kappa$$

where $\alpha_{\kappa \setminus \{hi\}}$ is the vector of all, except agent hi 's, actions. Each Pareto optimal strategy $\bar{s}_{\kappa} = (\bar{x}_{\kappa}^*, \bar{\alpha}_{\kappa}^*) \in \bar{S}^{\kappa}$ is the solution to a problem of the form (4.5.1)-(4.5.4). The entire set of Pareto optimal strategies is found by varying \bar{W}^{hi} , over all feasible values of W^{hi} , for all $hi \neq 11$.

The principal-agent models described in (1.2.2) and (1.2.4) in Chapter 1 are a special case of (4.5.1)-(4.5.4), and are obtained by imposing the following restrictions. Let $r=1$. Assigning to agent 11 the role of principal, α_{11}^{11} is a constant, so that agent 11 does not take an active part in altering the probabilities of the states. Therefore, agent 21 does not demand any actions from agent 11. Further, agent 21 has no claim over the endowment, receiving a payment c_j from the total endowment, and leaving agent 11 with a net return of $w_j^{11} + w_j^{21} - c_j$, $j=1,2$. Thus, agent 21 has no bargaining power in determining the precise nature of the compensation. Thirdly, the conventional principal-agent approach considers only one

member of \hat{S}^{κ} (if actions are enforceable) or $\hat{S}^{*\kappa}$ (if actions are unenforceable). The last two restrictions are particularly limiting, as they assume away the question of what contracts result from bargaining among agents. The analysis below investigates this issue, using the core as the solution concept.

It is well-known that the presence of public goods creates problems for a definition of the core. It requires a theory of the way in which the complementary coalition $-\theta$ reacts, when a coalition θ attempts to block a potential contract for the entire group κ . Rosenthal (1971) and Richter (1974) suggest that, given a potential blocking coalition, the complementary coalition might choose a contract which is group rational. But, this requires a definition of the core of the group $-\theta$. The problem is resolved by using an inductive approach to define the core.⁵ Suppose that, in response to a blocking coalition, $-\theta$ suggests a contract for its own members. A coalition, φ of $-\theta$ may attempt to block that contract, in which case a definition of the core is required for the now smaller complementary coalition $-(\theta+\varphi)$; and so on, until the complementary coalition reduces to one agent, for whom a core is easily defined.

4.6 A DEFINITION OF THE CORE

Edgeworth, Debreu and Scarf formulated a theory of the core for a private goods economy. Their definition of the core is now extended to the economy of section 4.3. This requires a characterization of core actions as well as core allocations. It is assumed that actions are observable, so that they can be written into a contract. The following assumption specifies the conditions under which an agent agrees to sign a contract which stipulates his action,^{6,7} so that the action is legally enforceable.

Assumption 4

Agent h_i 's action a^{h_i} is enforceable by the coalition θ if and only if $h_i \in \theta$.

Thus, agents who belong to a coalition θ are willing to sign a contract, written by the coalition for its own members, specifying their actions (and allocations). The reason for the willingness of agents in θ to sign some such contract is that, if actions are observable, there always exists a contract stipulating actions of all agents in the coalition, which is Pareto-superior to a contract which does not stipulate any actions. Formally, if actions are not written into a contract, then the maximization problem of the coalition contains

incentive-compatibility constraints of the form in (4.5.4) for all agents in θ , which need not be satisfied otherwise. The intuition is provided by the principal-agent literature, namely that, when actions are enforced, a better risk-sharing arrangement is possible.

On the other hand, agents who do not belong to θ do not share risks with agents in θ , and hence do not have an incentive to allow that coalition to enforce their actions.

When the model is interpreted as a generalization of the principal-agent model, Assumption 4 can be seen as an assumption about the extent of moral hazard in the economy. Since contracts in the core involve cooperation by all agents, such contracts do not exhibit moral hazard. Likewise, there is no moral hazard within coalitions. However, a coalition may choose actions that are undesirable from the viewpoint of agents not in the coalition (and vice versa). In this way the possibility of moral hazard determines the success of blocking coalitions, and this determines which contracts are in the core.⁸

Assumption 4 implies that a strategy $s_\theta = (x_\theta, \alpha_\theta, \alpha_{-\theta})$ can be implemented by a coalition θ only if the complementary coalition willingly chooses the actions $\alpha_{-\theta}$. In determining whether a strategy s_κ can be blocked, the coalition must therefore consider the choices of the complementary coalition. As discussed in section 4.5 this requires an assumption about

the behavior of the complementary coalition. The definition of the core below assumes that, given the formation of any coalition, its complementary coalition chooses a group rational strategy; that is, a strategy from the core of the group $-\theta$, given the actions of the coalition θ .

The core is now defined.⁹

Definition 1

(a) In a single agent economy $\hat{s} = (\hat{x}, \hat{\alpha})$ is a contract in the core if and only if $\hat{x} \leq w$ (so that the allocation is feasible), $\hat{\alpha} \in \{\underline{\alpha}, \bar{\alpha}\}$ (so that the action is feasible), and $(\hat{x}, \hat{\alpha})$ maximizes $W(x, \alpha)$.

(b) Let $1 < q < 2r$ and assume that the core for an economy with q agents has been defined. Then

- (i) For any economy with $2r$ agents, $\hat{s}_\theta = (\hat{x}_\theta, \hat{\alpha}_\theta, \hat{\alpha}_{-\theta})$ is a strategy which θ is able to implement if and only if
- (1) $\sum_\theta \hat{x}^{hi} \leq \sum_\theta w^{hi}$;
 - (2) $\alpha^{hi} \in \{\underline{\alpha}, \bar{\alpha}\} \forall hi \in \theta$; and
 - (3) any strategy $s'_{-\theta} = (x_{-\theta}, \alpha_\theta, \alpha_{-\theta})$ which belongs to the core of the subeconomy Γ/\hat{s}_θ satisfies $\alpha_{-\theta} = \hat{\alpha}_{-\theta}$.¹⁰
- (ii) θ blocks s_κ if and only if there exists a strategy \hat{s}_θ such that

- (1) θ is able to implement \hat{s}_θ ; and
- (2) \hat{s}_θ is preferred to s_κ by all agents in θ , and strictly preferred by some agent in θ .

(iii) s_{κ} is core of an economy with $2r$ agents if and only if

$$(1) \sum_{\kappa} x^{hi} \leq \sum_{\kappa} w^{hi};$$

$$(2) \alpha^{hi} \in \{\underline{\alpha}, \bar{\alpha}\} \quad \forall hi \in \kappa; \text{ and}$$

$$(3) \nexists \theta \in \Theta \text{ which blocks } s_{\kappa}.$$

The definition works as follows. Suppose that the agents in a group κ are considering whether to sign a contract s_{κ} . If a coalition θ decides not to engage in exchange with the group $-\theta$, then the latter suggests allocations and actions for its members, taking as given the actions of agents in θ . If no subgroup within $-\theta$ is dissatisfied with these allocations and actions, and if the actions are consistent with θ 's strategy (condition bi3); and θ 's suggested allocations and actions for its own members are feasible (bi1 and bi2), then θ 's strategy can be implemented by θ . If that strategy is also (strictly) preferred to the original suggestion s_{κ} , then s_{κ} is blocked by θ (bii). If, on the other hand, no coalition is able to find such a blocking strategy (biii3), then, as long as s_{κ} is feasible (biii1 and biii2), s_{κ} belongs to the core of the group κ . It might therefore be written into a contract, and observed.

Implicit in the above definition of the core is that coalitions exhibit Nash behavior. Given the actions of θ and its own endowment, the complementary coalition $-\theta$ chooses actions $\alpha_{-\theta}$ for itself. Likewise, the coalition θ takes as

given the choices of the complementary coalition and its own endowment when choosing its own actions α_θ .

The following sections derive properties of the core. To simplify the notation it is assumed that agents of different types have identical utility functions. However, all results hold for any utility functions satisfying the conditions in section 4.3. All proofs assume that actions are feasible $(\alpha^h_i \in \{\underline{\alpha}, \bar{\alpha}\} \forall h_i)$. The "r-core" denotes the core of an economy with r (identical) pairs of agents.

4.7 THE CORE AND PARETO OPTIMALITY

In the private goods model of Debreu and Scarf it is shown that a core allocation is Pareto optimal. If an allocation is suboptimal, then it is blocked by the coalition of all agents. This is a property also of the present model:

Lemma 1

If $s = (x_\kappa, \alpha_\kappa) \in r\text{-core}$, any r , then s_κ is Pareto optimal.

Proof: Let s_κ be suboptimal, and let $\hat{s}_\kappa = (\hat{x}_\kappa, \hat{\alpha}_\kappa)$ be a Pareto optimal strategy. Consider $\theta = \kappa$. Because \hat{s}_κ is feasible and $-\theta$ is empty, θ is able to implement \hat{s}_θ . Further, \hat{s}_θ is preferred to s_κ by all agents in θ and strictly preferred by some agent in θ . Hence, θ blocks s_κ , which contradicts the

assumption that $s_{\kappa} \in r\text{-core}$. ■

The solution to a problem of the form (4.5.1)-(4.5.3) is characterized by perfect risk-sharing (see, for instance, Borch, 1962). Further, given the functional form of π , a Pareto optimal strategy stipulates identical actions for all agents. Thus, by Lemma 1 all core contracts stipulate perfect risk-sharing and identical actions. Formally, given well-behaved preferences, a core contract s_{κ} satisfies

$$(4.7.1) \quad MRS(x_{\alpha_{\kappa}}^{hi}, \alpha_{\kappa}) = MRS(x_{\alpha_{\kappa}}^{kl}, \alpha_{\kappa}) \quad \forall hi, kl \in \kappa \quad \forall \alpha_{\kappa}$$

$$(4.7.2) \quad \alpha_{\kappa}^{hi} = \alpha_{\kappa}^{kl} \quad \forall hi, kl \in \kappa$$

Diagrammatically, in an economy with one agent of each type, Lemma 1 eliminates from the core all strategies stipulating allocations not on the contract curve. Further, since strategies with mixed actions are suboptimal, indifference curves parameterized by mixed actions are not relevant.

4.8 THE CORE OF AN ECONOMY WITH ONE PAIR OF AGENTS

In the model of Debreu and Scarf it is also readily shown that if, for some agent, an allocation is not individually rational (that is, it is not at least as good as

his endowment), then it is blocked by that agent. In the model of section 4.3 an agent will block a suggested contract if it is not as good as what he can achieve without any cooperation. This requires an analysis of what an agent can achieve in autarky when there is no exchange of the good. Consider an economy with one pair of agents. Assumption 5 below limits the class of agents' characteristics, and, by eliminating from the set of Pareto optimal strategies those which are not individually rational, allows a description of contracts in the 1-core (Lemma 2).

To state the assumption and characterize the 1-core, it is convenient to define Pareto optimal allocations u, v, y and z (see Figure 4.14.3) satisfying:

$$\begin{aligned} W(u^{11}, \underline{\alpha}_\kappa) &= W(w^{11}, \underline{\alpha}_\kappa) \\ W(v^{11}, \bar{\alpha}_\kappa) &= W(w^{11}, \bar{\alpha}_\kappa) \\ W(y^{21}, \underline{\alpha}_\kappa) &= W(w^{21}, \underline{\alpha}_\kappa) \\ W(z^{21}, \bar{\alpha}_\kappa) &= W(w^{21}, \bar{\alpha}_\kappa) \end{aligned}$$

Assumption 5

The expected utility functions and endowments of agents 11 and 21 satisfy

$$\begin{aligned} (a) \quad W(w^{h1}, \bar{\alpha}_\kappa) &> W(w^{h1}, \underline{\alpha}_\kappa) \quad h=1,2 \\ (b) \quad W(w^{21}, \bar{\alpha}_\kappa) &> W(x^{21}, \underline{\alpha}_\kappa) \quad \forall x^{21} \text{ such that } x_j^{21} \leq u_j^{21} \quad j=1,2 \end{aligned}$$

Part (a) states that in the absence of exchange of the

good, $\bar{\alpha}_\kappa$ is a Nash equilibrium. Hence $s_{\{h1\}}^\circ = (w^{h1}, \bar{\alpha}_\kappa)$ is a strategy which $\theta = \{h1\}$ is able to implement. For instance, if $h=1$, then the only strategy in the core of $r/s_{\{11\}}^\circ$ is $(w^{21}, \bar{\alpha}_\kappa)$, which satisfies $\alpha_{\{21\}} = \bar{\alpha}_{\{21\}}$.

Although $\bar{\alpha}_\kappa$ is a Nash equilibrium in autarky, exchange of the good alters agents' incentives to maintain those actions. Risk-sharing provides agent 21 with a greater incentive to choose $\bar{\alpha}$ (since risk-sharing makes state 2 more, and state 1 less desirable compared with the endowed allocation); but for agent 11 risk-sharing reduces the incentive to choose $\bar{\alpha}$. To maintain agent 11's choice of $\bar{\alpha}$ it is sufficient that agent 21 threaten him with autarky, which removes from agent 11 the benefits from risk-sharing. The incentive to carry out such a threat is contained in part (b).

In terms of the farmers' example, Assumption 5a states that, if there is only one pair of farmers, and no trade occurs to spread the risks associated with the yield, then if one farmer sprays his orchard, the other prefers spraying to not spraying his own orchard. However, if apples are traded across the high and low yield states, the type 1 farmer's fortunes become less dependent on a high yield, so that his incentive to continue spraying diminishes. Assumption 5b implies that the type 2 farmer's gain from a healthy orchard is sufficiently large relative to his gain from risk-sharing, that he is

willing to refuse trade with the type 1 farmer, if the latter suggests a contract specifying that spraying is not required. Thus, if the type 1 farmer wants to reduce his exposure to the risk of a low yield, he must agree to spray his orchard.

Lemma 2 characterizes contracts in the 1-core, namely those which are individually rational as well as Pareto optimal:

Lemma 2

If $s_{\kappa} = (x_{\kappa}, \alpha_{\kappa}) \in 1\text{-core}$ then

(a) $\alpha_{\kappa} = \bar{\alpha}_{\kappa}$; and

(b) $v_j^{11} \leq x_j^{11} \leq z_j^{11} \quad j=1,2$

Proof: (a) Let $s_{\kappa} = (x_{\kappa}, \alpha_{\kappa})$. Given that x_{κ} and u are both Pareto optimal allocations (using Lemma 1 and the definition of u), either (i) $x_j^{11} < u_j^{11} \quad j=1,2$ or (ii) $x_j^{11} \geq u_j^{11} \quad j=1,2$.

If (i) holds, then $\theta=\{11\}$ blocks s_{κ} with $\dot{s}_{\{11\}}$.

If (ii) holds, then $\theta=\{21\}$ blocks s_{κ} with $\dot{s}_{\{21\}}$ using

Assumption 5b.

(b) Let $s_{\kappa} = (x_{\kappa}, \bar{\alpha}_{\kappa})$. Then, by Assumption 5,

if $x_j^{11} < v_j^{11} \quad j=1,2$, then $\theta=\{11\}$ blocks s_{κ} with $\dot{s}_{\{11\}}$; and

if $x_j^{11} > z_j^{11} \quad j=1,2$, then $\theta=\{21\}$ blocks s_{κ} with $\dot{s}_{\{21\}}$. ■

Thus, in an economy with one pair of agents core contracts are characterized by perfect risk-sharing and high effort from both

agents.

4.9 THE CORE WHEN THERE ARE r PAIRS OF AGENTS

When the economy is replicated, agents in a complementary coalition, no matter how small, can significantly alter the welfare of agents in the coalition, because of their potential effect on the value of π . If a coalition desires $\bar{\pi}$, all agents in the complementary coalition must choose $\bar{\alpha}$. This distinguishes the model from the conventional Debreu-Scarff model in which any agent's influence on outcomes becomes smaller as the economy grows.

When there is one pair of agents, a choice of $\bar{\alpha}$ by a single agent coalition induces the other agent to also choose $\bar{\alpha}$. Under risk-sharing the benefits derived by agent 11 are sufficient to induce a choice of $\bar{\alpha}$, if that is needed to persuade agent 21 to engage in risk-sharing. The incentive mechanism of this model therefore works as follows. The agent with the greatest incentive to shirk is motivated to choose a favorable action by the opportunity it brings to reduce the exposure to risk. An issue considered in this section is whether replication, by creating new opportunities for risk-sharing, creates incentives for any agent to choose $\underline{\alpha}$. It is shown that, whatever the size of the economy, agents agree only on actions found in 1-core contracts. Thus, the incentive to

choose $\bar{\alpha}$ are maintained under replication. This result is used to show that identical agents are treated equally in the core, and that the core shrinks to a competitive contract as the economy grows. When Debreu and Scarf prove shrinkage of the core for a private goods economy, they first show that, whatever the size of the economy, agents of the same type receive identical consumption bundles. Given this, they show that the core shrinks to a competitive allocation as the economy grows large. In the present model, it does not seem possible to prove the equal treatment property for an economy with r pairs of agents without proving that the core shrinks as the economy grows from 1 to $r-1$ pairs. The results are therefore extended as follows. It is shown that there is equal treatment in the 2-core, and that the 2-core is contained in the 1-core. Effort by 2 pairs of agents is therefore high, whatever strategy is actually chosen. These results are then extended by induction to the r -core.

Lemma 3

If $s_{\kappa} = (x_{\kappa}, \alpha_{\kappa}) \in 2\text{-core}$, then $x_{\kappa}^{hi} = x_{\kappa}^{hk}$ and $\alpha_{\kappa}^{hi} = \alpha_{\kappa}^{hk}$ $h, i, k = 1, 2$.

Proof: If $s_{\kappa} \in 2\text{-core}$, then actions are identical by Lemma 1 and (4.7.2). Let $\alpha_{\kappa}^{h1} = \alpha_{\kappa}^{h2}$ $h = 1, 2$ and $x_{\kappa}^{k1} = x_{\kappa}^{k2}$ $k = 1$ or 2 .

W.l.o.g. let $\theta = \{11, 21\}$ be the worse-treated pair of agents.

Given α_κ , C_1 (in Appendix 1, section 4.12) implies that there exists a feasible \hat{x}_θ such that

$$(4.9.1) \quad W(\hat{x}_\theta^{\text{hi}}, \alpha_\kappa) \geq W(x_\kappa^{\text{hi}}, \alpha_\kappa) \quad \forall h_i \in \theta \text{ with } > \text{ for some } h_i \in \theta.$$

Consider $\hat{s}_\theta = (\hat{x}_\theta, \hat{\alpha}_\kappa)$, $\hat{\alpha}_\kappa = \alpha_\kappa$ (\hat{s}_θ and s_κ stipulate the same vector of actions). θ is able to implement \hat{s}_θ if core strategies $s'_{-\theta} = (x_{-\theta}, \hat{\alpha}_\theta, \alpha_{-\theta})$ in the subeconomy r/\hat{s}_θ satisfy $\alpha_{-\theta} = \hat{\alpha}_{-\theta}$.

Let $\hat{\alpha}_\kappa = \alpha_\kappa$. Then $\alpha_{-\theta} = \alpha_{-\theta}$ as required.

Let $\hat{\alpha}_\kappa = \alpha_\kappa$. Then, because r/\hat{s}_θ is like an economy with $r=1$, $-\theta$ chooses a strategy satisfying $\alpha_{-\theta} = \alpha_{-\theta}$ by Lemma 2a.

Hence, θ is able to implement \hat{s}_θ . Then with (4.9.1) θ blocks s_κ , so that contracts in the 2-core stipulate identical allocations within types of agents. ■

As in the Debreu-Scarfe theory, the equal treatment property implies that the theory can be written with reference to a representative pair of agents $(1i, 2i)$. Lemma 4 shows that contracts belonging to the 2-core do not involve allocations and actions that are not in the 1-core:

Lemma 4

Let $r=2$. Let a strategy $s_\kappa = (x_\kappa, \alpha_\kappa)$ be such that $s = (x^{1i}, x^{2i}, \alpha^{1i}, \alpha^{2i}) \notin 1\text{-core } i=1, 2$. Then $s_\kappa \notin 2\text{-core}$.

Proof: Since $s \notin 1\text{-core} \exists \varphi \subseteq \{11, 21\} = \psi$ which blocks s with a strategy \dot{s}_φ . Either

- (i) $\varphi = \{11, 21\}$ and $\dot{s}_\varphi = (\dot{x}_\varphi, \dot{\alpha}_\psi)$ or
(ii) $\varphi = \{h1\}$ and $\dot{s} = \dot{s}_{\{h1\}}$ $h=1$ or 2 .

Consider (i). Let $\theta = \kappa$ and let \hat{s}_θ be such that $\hat{x}^{h1} = \hat{x}^{h2} = \dot{x}^{h1}$ and $\hat{\alpha}^{h1} = \hat{\alpha}^{h2} = \dot{\alpha}^{h1}$ $h=1, 2$. θ is able to implement \hat{s}_θ , because \dot{s}_φ is feasible, so that \hat{s}_θ is feasible, and because $-\theta$ is empty. Further, for $h, i=1, 2$

$$\begin{aligned} W(\hat{x}^{hi}, \hat{\alpha}_\kappa) &= W(\dot{x}^{h1}, \dot{\alpha}_\psi) \\ &\geq W(x^{h1}, \alpha_\psi) > \text{for } h=1 \text{ or } 2 \\ &= W(x^{hi}, \alpha_\kappa). \end{aligned}$$

Hence, θ blocks s_κ .

Consider (ii). Let $\theta = \{h1, h2\}$ and $\hat{s}_\theta = (w_\theta, \bar{\alpha}_\kappa)$.

Since $W(w^{k\ell}, \bar{\alpha}_\kappa) > W(w^{k\ell}, \alpha_\kappa)$ $k \neq h$ $\ell=1, 2$ (Assumption 5a), $-\theta$ chooses $\bar{\alpha}_{-\theta}$, given $\bar{\alpha}_\theta$. Hence, θ is able to implement \hat{s}_θ .

Further, for $i=1, 2$

$$\begin{aligned} W(w^{hi}, \bar{\alpha}_\kappa) &= W(w^{h1}, \bar{\alpha}_\psi) \\ &> W(x^{h1}, \alpha_\psi) \\ &= W(x^{hi}, \alpha_\kappa). \end{aligned}$$

Hence, θ blocks s_κ . ■

Given Lemma 2a, Lemma 4 implies that contracts in the 2-core

always stipulate high effort.

It is now possible to prove that there is equal treatment in the core for any number of replications:

Proposition 1

If $s = (x_{\kappa}, \alpha_{\kappa}) \in r\text{-core}$, then $x_{\kappa}^{hi} = x_{\kappa}^{hk}$ and $\alpha_{\kappa}^{hi} = \alpha_{\kappa}^{hk} \quad \forall i, k \in \{1, \dots, r\}$
 $h=1, 2$.

Proof: The proposition holds when the number of pairs p equals 2, by Lemma 3. Assume that the proposition holds when $3 \leq p \leq r-1$. It can then be shown that the $(r-1)\text{-core} \subseteq 1\text{-core}$ using an argument analogous to the proof of Lemma 4. Hence, if $s = (x_{\kappa}, \alpha_{\kappa}) \in (r-1)\text{-core}$, then $s_{\psi} \in 1\text{-core}$ where $\psi = \{1i, 2i\}$ $i=1, \dots, r-1$, so that $\alpha_{\kappa} = \bar{\alpha}_{\kappa}$ by Lemma 2a. It can then be shown that the proposition holds when $p=r$, using an argument analogous to the proof of Lemma 3. ■

Given equal treatment in the $r\text{-core}$, Lemma 4 can be extended to the $r\text{-core}$, using an argument analogous to the proof of Lemma 4:

Lemma 5

Let $s = (x_{\kappa}, \alpha_{\kappa})$ be such that $(x_{\kappa}^{1i}, x_{\kappa}^{2i}, \alpha_{\kappa}^{1i}, \alpha_{\kappa}^{2i}) \notin 1\text{-core} \quad i=1, \dots, r$.
 Then $s \notin r\text{-core}$.

Lemma 5 implies

Lemma 6

If $s_{\kappa} = (x_{\kappa}, \alpha_{\kappa}) \in r\text{-core}$, then $\alpha_{\kappa} = \bar{\alpha}_{\kappa}$.

Proof: Immediate from Lemmas 5 and 2a. ■

Now consider an economy, identical to the economy described in section 4.3, but with the value of π exogenously given by $\bar{\pi}$. Let $\bar{W}: X^{hi} \rightarrow R$ be defined by

$$\bar{W}(x^{hi}) = (1-\bar{\pi})U(x_1^{hi}) + \bar{\pi}U(x_2^{hi}) \quad \forall hi \in \kappa$$

U as defined in section 4.3.

Let $\tilde{x} = (\tilde{x}^{1i}, \tilde{x}^{2i})$ $i=1, \dots, r$ denote the bundle, which a representative pair of agents would consume at an equilibrium in a competitive market for state-contingent claims against the private good, given that $\bar{\pi}$ is the probability of state 2. \tilde{x} is illustrated in Figure 4.14.4. Let $\tilde{x}_{\kappa} = (\tilde{x}^{11}, \dots, \tilde{x}^{2r})$. Formally,

Definition 2

A competitive equilibrium in a market for state-claims against the private good, given that $\bar{\pi}$ is the probability of state 2, is a system of prices $(\tilde{p}_1, \tilde{p}_2)$ and an allocation $(\tilde{x}^{11}, \dots, \tilde{x}^{2r})$ such that

(a) \tilde{x}^{hi} is a solution to

$$\text{Max}_{x^{hi}} \bar{W}(x^{hi})$$

$$\text{s.t.} \quad \tilde{p}_1 x_1^{hi} + \tilde{p}_2 x_2^{hi} \leq \tilde{p}_1 w_1^{hi} + \tilde{p}_2 w_2^{hi} \quad \forall i \in \kappa; \text{ and}$$

$$(b) \quad \sum_{\kappa} \tilde{x}^{hi} = \sum_{\kappa} w^{hi}.$$

Proposition 2 shows that the set of core contracts shrinks to the contract $\tilde{s} = (\tilde{x}_{\kappa}, \bar{\alpha}_{\kappa})$ as the economy grows large:

Proposition 2

If $s_{\kappa} \in r\text{-core} \forall r$, then $s_{\kappa} = \tilde{s}$.

Proof: If $s_{\kappa} \in r\text{-core}$ any r , then $\alpha_{\kappa} = \bar{\alpha}_{\kappa}$ by Lemma 6, so that $\pi(\alpha_{\kappa}) = \bar{\pi}$. Then, using C_2 (see Appendix 1) $x_{\kappa} = \tilde{x}_{\kappa}$. ■

4.10 EXISTENCE OF THE CORE

If Assumption 5b is strengthened to Assumption 6 below, it may be shown that the core, as defined in section 4.6, exists for the model of section 4.3. The argument extends that used by Debreu and Scarf, who show that the competitive allocation in a private goods economy is always contained in the core. In the present model, \tilde{s} is always contained in the core.

Let t be a Pareto optimal allocation satisfying

$$\frac{w_2^{1i} - t_2^{1i}}{t_1^{1i} - w_1^{1i}} = \text{MRS}(w^{1i}, \underline{\alpha}_\kappa)$$

In words, for a representative pair $(1i, 2i)$ of agents, t is the allocation which perfectly shares risks, and is just affordable to agent $1i$, given a low probability (π) that state 2 will occur, and a relative price of the good in state 1 equal to agent $1i$'s marginal rate of substitution at his endowment (see Figure 4.14.3). To prove existence of the core (Proposition 3) Assumption 5b is replaced by:

Assumption 6

$W(w^{21}, \bar{\alpha}_\kappa) > W(x^{21}, \underline{\alpha}_\kappa) \quad \forall x^{21}$ such that $x_j^{21} \leq t_j^{21} \quad j=1, 2$.

Proposition 3

$\tilde{s} \in r\text{-core} \quad \forall r$.

Proof: Let $\tilde{s} \in r\text{-core}$, some r . Then $\exists \theta$ which blocks \tilde{s} with the strategy \hat{s}_θ . Either (a) $\hat{s}_\theta = (\hat{x}_\theta, \bar{\alpha}_\kappa)$ or (b) $\hat{s}_\theta = (\hat{x}_\theta, \underline{\alpha}_\kappa)$.

Consider (a). By C_3 (in Appendix 1), given $\alpha_\kappa = \bar{\alpha}_\kappa$ and hence $\pi(\alpha_\kappa) = \bar{\pi}$, there does not exist an allocation x_θ which can be used to block \tilde{x}_κ . Hence, there does not exist a strategy $(\hat{x}_\theta, \bar{\alpha}_\kappa)$ which can be used to block $(\tilde{x}_\kappa, \bar{\alpha}_\kappa)$.

Consider (b). Either

- (i) θ contains only type 1 agents; or
- (ii) θ contains at least one type 2 agent.

Consider (i) Since all agents in θ are identical, $\hat{x}_\theta = w_\theta$.

Moreover, if θ blocks \tilde{s} , then

$$\begin{aligned} W(w^{1i}, \alpha_\kappa) &\geq W(\tilde{x}^{1i}, \bar{\alpha}_\kappa) \quad \forall 1i \in \theta \text{ with } > \text{ for some } 1i \in \theta. \\ &> W(w^{1i}, \bar{\alpha}_\kappa) \text{ since gains from risk-sharing} \end{aligned}$$

exist. This contradicts Assumption 5a.

Consider (ii). W.l.o.g. let θ contain agent 21. If θ blocks \tilde{s} , then

$$W(\hat{x}^{21}, \alpha_\kappa) \geq W(\tilde{x}^{21}, \bar{\alpha}_\kappa) > W(w^{21}, \bar{\alpha}_\kappa)$$

since gains from risk-sharing exist. Hence, using Assumption

6, $\hat{x}_j^{21} > t_j^{21}$ $j=1,2$. But, then at least one type 1 agent, say agent 11, must buy the good in state 1 from agent 21 at a price exceeding $MRS(w^{11}, \alpha_\kappa)$, so that

$$\begin{aligned} W(\hat{x}^{11}, \alpha_\kappa) &< W(w^{11}, \alpha_\kappa) < W(w^{11}, \bar{\alpha}_\kappa) \text{ by Assumption 5a} \\ &< W(\tilde{x}^{11}, \bar{\alpha}_\kappa) \end{aligned}$$

since gains from risk-sharing exist. Thus, if θ is able to implement \hat{s}_θ , this strategy can not also be preferred by all agents in θ . For both (a) and (b) the assumption that there exists a coalition which blocks \tilde{s} therefore leads to a contradiction. ■

4.11 CONCLUSION

A number of authors have observed that the existence of public goods creates interdependencies, which need not be considered in a theory of the core for a private goods economy. An economy with moral hazard is an example of a public goods economy, since agents' actions may be interpreted as public goods. The definition of the core and Assumptions 4-6 show that to accomodate those actions the Debreu-Scarf theory of the core must be extended in various directions. In particular, a specification is needed of

- the behavior of agents who are left out of a coalition (Definition 1b13);
- the extent of moral hazard (Assumption 4);
- the effect of replication, which adds more agents, and hence more actions (captured in the functional form of π).

For a particular class of moral hazard economies it is shown that the core exists, that core contracts specify identical treatment of identical agents, and that the core shrinks as the economy grows large. The limiting contract is the one that prevails in a competitive market, in which agents trade state-claims against the private good, taking as given that $\bar{\pi}$ is the probability of the favorable state.

The result that the core exists implies that, for the class of economies under consideration, agents will contract to

avoid moral hazard. Specifically, they will not split into multiple coalitions, each confronted with the possibility of moral hazard on the part of other coalitions. The analysis shows that agents will come to a collective agreement about the optimal set of actions, and will carry out those actions.

4.12 APPENDIX 1

Let \tilde{x}_κ be defined as in section 4.9. The following existing results about the core in a private goods economy then apply (see, for instance, Varian, 1984):

C_1 : If x_κ satisfies $x^{hi}_\kappa \neq x^{hk}_\kappa$ some h and $i \neq k$, then the coalition $\theta = \{\text{worst-treated pair of agents}\}$ blocks x_κ .

C_2 : Given $\bar{\pi}$, if $x_\kappa \in r\text{-core } \forall r$, then $x_\kappa = \tilde{x}_\kappa$.

C_3 : Given $\bar{\pi}$, $\tilde{x}_\kappa \in r\text{-core } \forall r$.

4.13 APPENDIX 2

The forcing contract of Spence and Zeckhauser (1971) and similar models has the following form:

$$c_i = \hat{c}_i \quad \text{iff} \quad \alpha = \hat{\alpha} \quad i=1,2$$

$$c_i = 0 \quad \text{iff} \quad \alpha \neq \hat{\alpha} \quad i=1,2$$

where c_i is the payment received by the agent in state i , and $(\hat{c}_1, \hat{c}_2, \hat{\alpha})$ is the optimal contract when actions can be contracted upon. By threatening to withhold the agent's payment, the principal induces him to take the assigned action.

In the model of section 4.3 the question arises how agents force actions onto one another, given that there is no outsider, such as a principal, who can punish a shirking agent by confiscating his payment. Agent $hi \in S$ may confiscate the

allocation of agent $k \in S$ if the latter shirks but agent h_i may also shirk! It is not possible to threaten every agent with confiscation of his allocation, unless the court (or other outside party) takes some of the endowment as punishment. However, ex post (all agents having shirked), it is preferable to keep the entire endowment within the group, so that no one may want to involve the court. The question is how the contract can give some agent an incentive to take the group to court whenever there is shirking (even if he himself shirks). It is shown below that if, having written the contract, agents choose actions under Nash conjectures, then there exists a forcing contract, which motivates all agents to choose the specified actions, yet allocates the total endowment to the group, regardless of any shirking.

Let \hat{x}^{h_i} and $\hat{\alpha}^{h_i}$ be the allocation and action assigned to agent $h_i \in \theta$. Let w denote the ex post total endowment; let θ^* denote the subset of agents in θ who choose $\hat{\alpha}^{h_i} \neq \hat{\alpha}^{h_i}$; and let $|\theta|$ denote the cardinal number of θ . Then the forcing contract is

$$(F1) \quad \text{If } \theta^* = \emptyset \quad \text{then} \quad x^{h_i} = \hat{x}^{h_i} \quad \forall h_i \in \theta$$

$$(F2) \quad \text{If } \theta^* \notin \{\emptyset, \theta\} \quad \text{then}$$

$$x^{h_i} = 0 \quad \forall h_i \in \theta^*$$

$$x^{h_i} = w / |\theta \setminus \theta^*| \quad \forall h_i \in \theta \setminus \theta^*$$

(F3) If $\theta^* = \theta$ then

$$x^{hi} = 0 \quad \forall hi \in \theta \setminus \{pq\} \text{ some arbitrary } pq \in \theta$$

$$x^{pq} = w$$

Under such a contract the total endowment is always consumed within the coalition. If no one shirks everyone receives the assigned allocation. If some, but not all, agents shirk, the total endowment is distributed among the non-shirking agents. (This is similar to the traditional principal-agent forcing contract, where the non-shirking party - namely, the principal - receives the total outcome.) If all agents shirk then one (chosen in any fashion and specified in the contract before the actions are carried out) receives the total endowment of the coalition.

All agents are then induced, in the following way, to carry out the assigned actions. Consider any agent hi . If all other agents carry out the assigned actions, then the best strategy for agent hi is to do likewise (by (F1) and (F2)). Thus, choosing the agreed upon actions by all agents is a Nash equilibrium. Now suppose that some agents shirk, while others do not. The best strategy of a given agent hi is to take the assigned action, to avoid losing his endowment (by (F2)). Therefore, a situation where a subgroup of agents shirks is not a Nash equilibrium. If all agents other than agent hi shirk,

then the best course is for agent h_i to not shirk, so that he can claim the entire endowment (by (F3)).

Proposition 4

There exists an efficient forcing contract for the coalition θ .

Proof: Consider the contract (F1)-(F3). The set of chosen actions must satisfy one of the following:

- (a) All agents choose the agreed upon actions;
- (b) A proper subset of agents shirks; or
- (c) All agents shirk.

It is shown that only the first constitutes a Nash equilibrium.

Let $\alpha^{kl} = \hat{\alpha}^{kl} \forall kl \in \theta \setminus \{h_i\}$. If agent h_i chooses $\hat{\alpha}^{h_i}$, then his expected utility is $W(\hat{x}^{h_i}, \hat{\alpha}_{\kappa}^{h_i}) > 0$ by (F1). Otherwise it is $-\alpha^{h_i} \leq 0$ by (F2). Hence, agent h_i chooses $\hat{\alpha}^{h_i}$. This is true for all $h_i \in \theta$, so that (a) is a Nash equilibrium.

Let $\alpha^{kl} \neq \hat{\alpha}^{kl} \forall kl \in \theta \notin \{\emptyset, \theta\} \quad kl \neq h_i$, and let $\alpha^{kl} = \hat{\alpha}^{kl} \forall kl \in \theta \setminus (\theta^* \cup \{h_i\})$. If agent h_i chooses $\hat{\alpha}^{h_i}$, then his expected utility is $W(w/|\theta \setminus \theta^*|, \alpha_{\kappa}^{h_i})$ by (F2). Otherwise it is $-\alpha^{h_i}$ by (F2). Hence, agent h_i chooses $\hat{\alpha}^{h_i}$. This is true for all $h_i \in \theta$, so that (b) is not a Nash equilibrium.

Let $\alpha^{kl} \neq \hat{\alpha}^{kl} \forall kl \in \theta \setminus \{h_i\}$ with $h_i \neq pq$ (pq as defined in (F3)). If agent h_i chooses $\hat{\alpha}^{h_i}$, then his expected utility is $W(w, \alpha_{\kappa}^{h_i})$ by (F2). Otherwise it is $-\alpha^{h_i}$ by (F3). Hence, agent h_i chooses

$\hat{\alpha}^{hi}$, so that (c) is not a Nash equilibrium.

The contract (F1)-(F3) therefore generates $\hat{\alpha}_\theta$ as a unique Nash equilibrium, and distributes the total endowment of θ within that coalition. ■

4.14

FIGURES

Figure 4.14.1

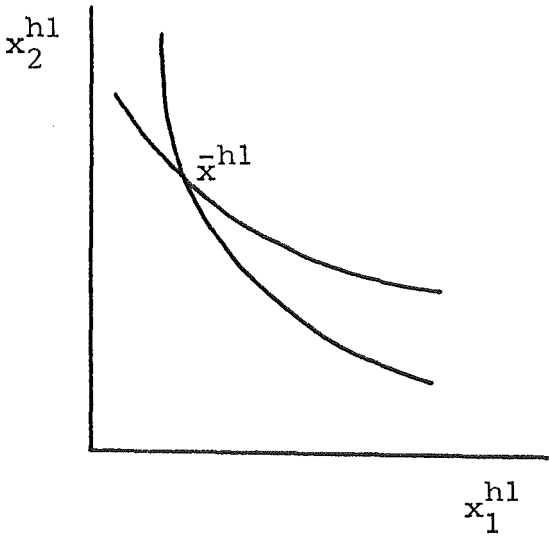


Figure 4.14.2

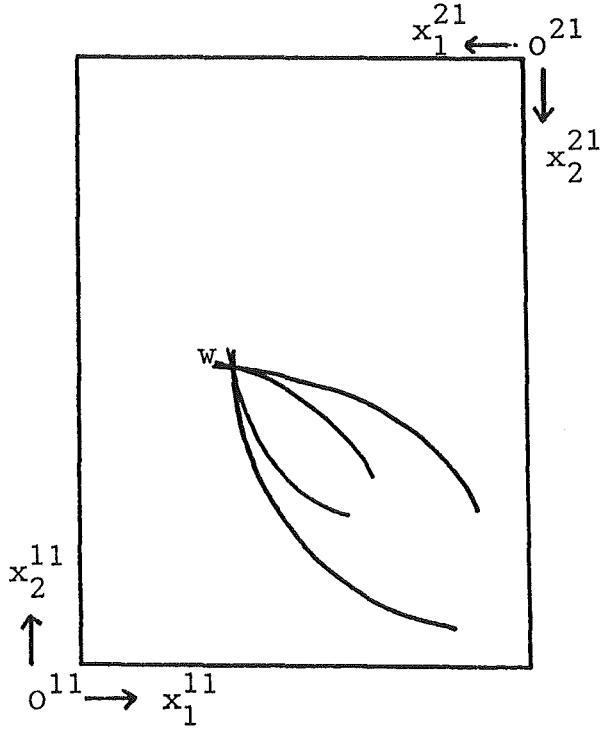
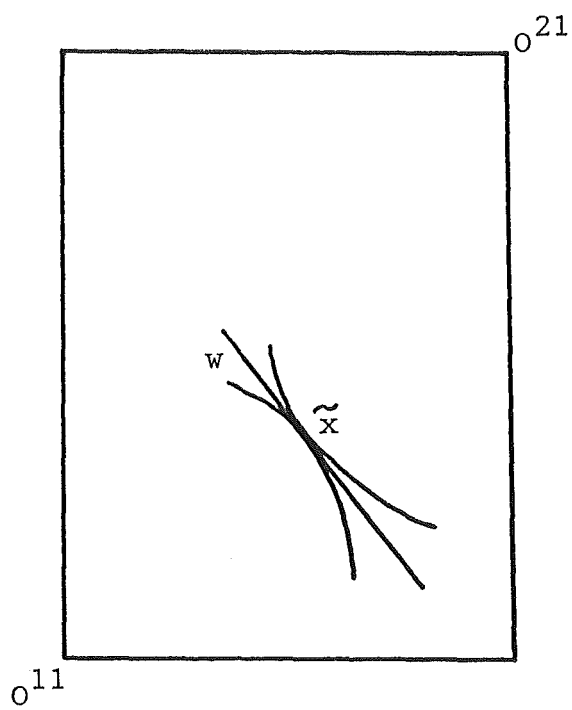
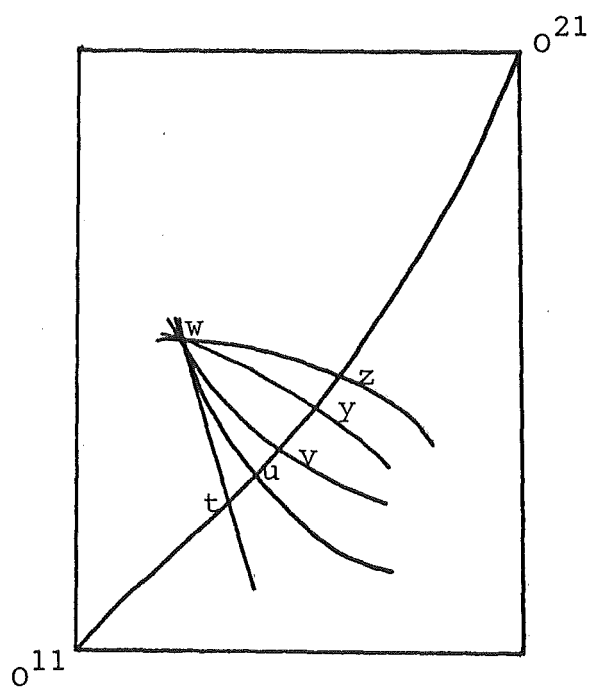


Figure 4.14.3



4.15 NOTES

1. Thus, the model is a generalization of Chapter 3, which allows the principal to choose an action, but does not allow the agent to bargain for a return above his reservation expected utility.
2. The uncertainty in this model is common to all agents - if state 1 occurs all agents receive a small endowment, and if state 2 occurs all receive a large endowment.
3. Since actions are observable, they can be contracted upon.
4. Since strategies not in the core never form the basis of a legally enforceable contract, the term "contract" will be used only to describe strategies which do belong to the core.
5. The use of induction was suggested by the definition of Coalition-proof Nash equilibrium by Bernheim and others (1987).
6. In principal-agent models, actions which are observable are also enforceable, because it is implicitly assumed that the principal and agent constitute a coalition. In this model, observability of actions is not sufficient for their enforceability, because agents do not necessarily belong to the same coalition.
7. In the conventional principal-agent model, if actions are enforced, the associated contract is called a "forcing" contract. It specifies which actions should be taken, and at

the same time provides incentives to the agent to take the specified actions. Appendix 2 in section 4.13 expands on the nature of the forcing contract in the conventional model and for the model of section 4.3.

8. Moral hazard is a more serious problem if actions are unobservable, and hence unenforceable within coalitions. In such a world contracts in the core are likely to exhibit moral hazard.

9. Although the definition uses the set of agents $\kappa = \{1, \dots, 2r\}$, it is not necessary to restrict the group to two types or an equal number of agents within types. However, those restrictions are necessary in the following analysis.

10. Since θ is non-empty, the subeconomy r/\hat{s}_θ contains fewer than $2r$ agents. Its core is therefore defined by the assumption in the stem of part b.

CHAPTER 5

INCENTIVES AND INSURANCE IN A PARTIAL EQUILIBRIUM
EDUCATION MODEL

5.1 INTRODUCTION

Schultz (1961), Becker (1962), Ben-Porath (1967), and Razin (1972a, 1972b) among others initiated a study of the way in which individuals make decisions about investments in education. Schultz, Becker, and more recently Levhari and Weiss (1974) and Eaton and Rosen (1980) observe that since there is generally imperfect information about such factors as abilities, the quality of schooling or the future return to skill, uncertainty plays an important role in those decisions. The analysis below extends this literature by introducing endogenous uncertainty. Specifically, the expected amount of human capital that results from education depends upon the graduation rate, which is affected by students' decisions about effort and the level of education.

One effect of uncertainty about the return to education is that the private sector is generally unwilling to provide consumer loans to finance education (for example, Schultz, 1961p.14). The possibility of moral hazard also contributes to this state of affairs (for example, Manning, 1985, p.18). As a

result governments often support education by way of loans or subsidies.

Using a two-period overlapping generations model, it is assumed that students have no access to private sector funds, but that the government makes available a sum of money during the period of study. It collects money when individuals become full-time workers in the following period in order to fund the following generation of students. Such a scheme may be interpreted as a loan facility, with students funding (part of) their consumption from loans, and repaying the loans when they begin full-time work (this is the interpretation adopted below); or as a subsidy paid to students, funded from taxes levied on full-time workers.

The provision of loans introduces a second issue - namely moral hazard - in the following way. Although students eventually repay the loans made available to them, the fact that repayments are always used to fund the following generation of students, may change students' incentives to study. Each dollar earned by a full-time worker is only partially consumed by himself, the remainder being paid to the creditor. Suppose that graduates make higher repayments than failures, so that an increase in student effort raises the expected repayment. Effort then generates a positive externality and, since students do not take this into account, they supply too little effort.

A consequence of the possibility of moral hazard is that individuals may not be able to buy private sector insurance against low future incomes. This problem is observed by Eaton and Rosen (1980) and by Manning (1985). It is shown below that the government, by appropriately specifying the terms of repayment of student loans, can insure the return to education. In particular, if a student who graduates and therefore earns a higher income during the rest of his life, faces a higher repayment than a student who fails, then the loan scheme reduces the gap between disposable skilled and unskilled income. (It is shown that such a scheme is always optimal, so that there is cause for moral hazard as identified in the preceeding paragraph.)

In terms of the principal-agent literature endogeneity of the education level adds a new dimension to the moral hazard problem. It means that the duration of the principal-agent relationship, which in the model below encompasses two periods, is endogenous.¹ Further, if, for a given level of effort, a higher education makes a student less likely to graduate, then (given some insurance of the return to education) education exerts a negative externality, because it lowers the government's funding for educating the next generation.² An issue considered below is how a shift in the control of effort from the government to students affects students' choice of education.

The analysis further considers the socially optimal level of insurance, given an exogenous government loan. Since the analysis does not solve for the optimal size of the loan, it should be interpreted as describing the second-best optima. The principal-agent literature suggests that for a given level of education, if the chances of graduating depend only on the level of effort, then a government which controls (does not control) effort should choose the terms of repayment to fully (partially) insure income. A question is whether this continues to hold when the level of education is a choice variable.

A third issue concerns the effect of a change in the population growth rate on the optimal repayments terms, the level of student effort and the level of education. This is investigated in a model in which the choice of education is endogenous, but does not affect the probability of graduating.

The analysis is arranged as follows. Section 5.2 describes a model of education. Section 5.3 studies the nature of education and the effect of population growth when the government controls student effort. This is repeated in section 5.4 when the government is unable to control effort. The results are summarized in section 5.5.

5.2 A MODEL OF EDUCATION³

Consider a partial equilibrium overlapping generations model in which each generation lives two periods. The model does not generate full equilibria, because it assumes away the response of relative wages to changes in the composition of the workforce.⁴ The typical individual is endowed with a unit of productive time⁵ in each period. An individual born in period t can use the productive time in that period to get an education and to work for an unskilled wage of $w_0(t)$. Let $\beta(t)$ be the amount of time allocated to education. Then $(1-\beta(t))w_0(t)$ is the individual's income.

It is assumed that there are no capital markets. It is taken as given that an education is feasible only with outside funding, that the government provides such finance by way of a loan L , and that from the point of view of the individual it is optimal to acquire an education. To simplify the analysis the amount L is exogenous. Consumption in period t by a newly born individual is therefore

$$(5.2.1) \quad C(t) = L + (1-\beta(t))w_0(t)$$

During period t the individual, as a student, engages in effort $\alpha(t)$. This is a fraction of the individual's leisure time.⁶

In period $t+1$ the individual works during his productive time. The wage depends on whether he fails or graduates. If he fails he continues to earn an unskilled wage $w_0(t+1)$. If he graduates he earns a skilled wage which is dependent on the level of education. Let $w(\beta(t), t+1)$ ⁷ denote the skilled wage in period $t+1$. Assume that it increases with the level of education at a diminishing rate:

Assumption 1

$$w'(\cdot) > 0, \quad w''(\cdot) < 0$$

with superscripts denoting derivatives.

In period $t+1$ the government requires repayment of the loans granted in period t . If a student fails in period t , then the government requires that $\phi_0(t+1)$ per dollar be repaid in period $t+1$; if he graduates then $\phi_1(t+1)$ per dollar must be repaid. If $\phi_1 > 1$ then there is an implicit interest payment. Net income and (in the absence of capital markets) consumption are therefore

$$(5.2.2) \quad C_0(t+1) = w_0(t+1) - \phi_0(t+1)L$$

if the student fails; or

$$(5.2.3) \quad C_1(t+1) = w(\beta(t), t+1) - \phi_1(t+1)L$$

if the student graduates.

The probability of graduating is $\pi(\alpha(t), \beta(t))$, being a function of the level of student effort, and the level of education. For a given level of education, more effort increases the chances of graduating, at a diminishing rate. On the other hand, given a level of effort, a higher education lowers the probability of graduating, at an increasing rate. Changes in effort do not affect the marginal effect of education on the probability of graduating:

Assumption 2

$$\begin{aligned} \pi^{\alpha}(\cdot) &> 0, & \pi^{\beta}(\cdot) &< 0 \\ \pi^{\alpha\alpha}(\cdot) &< 0, & \pi^{\beta\beta}(\cdot) &< 0, & \pi^{\alpha\beta}(\cdot) &= 0 \end{aligned}$$

The individual derives utility from consumption and disutility from effort, and is risk-averse. Let U and V be utility functions over consumption and effort, satisfying

Assumption 3

$$U'(\cdot) > 0, \quad U''(\cdot) < 0, \quad V'(\cdot) = -1$$

Let $r(t)$ be the discount factor of an individual born in period t . The discounted expected utility of such a person is

$$(5.2.4) \quad W(t) = U(C(t)) - \alpha(t) +$$

$$r(t) \left[(1 - \pi(\alpha(t), \beta(t))) U(C_0(t+1)) + \pi(\alpha(t), \beta(t)) U(C_1(t+1)) \right]$$

with the consumption levels given by (5.2.1)-(5.2.3). The individual is an expected utility maximizer.

The variables to be determined are the level of education, the level of effort and the repayment terms. The level of education $\beta(t)$ is always determined by the individual, the repayment terms $\phi_0(t)$ and $\phi_1(t)$ are always determined by the government, and the level of effort $\alpha(t)$ is controlled either by the individual or the government. If a change in effort affects the government's ability to finance future loans, and hence social as well as private welfare, it may be expected that society is better off if the government specifies what effort students take.

The repayment terms are chosen by the government under the constraint that in each period total repayments by full-time workers must be sufficient to pay for the loans to students in the same period. As long as there are sufficiently many students, the actual total repayment equals the expected total repayment. Further, if the population grows at rate n , then there are always $1+n$ times as many students as full-time workers, so that the government's budget constraint in period $t+1$ is

$$(1+n)S(t)L \leq (1-\pi(t))S(t)\phi_0(t+1)L + \pi(t)S(t)\phi_1(t+1)L$$

where $S(t)$ is the number of students in period t and

$\pi(t) = \pi(\alpha(t), \beta(t))$; or

$$(5.2.5) \quad 1+n \leq (1-\pi(t))\phi_0(t+1) + \pi(t)\phi_1(t+1)$$

The analysis is confined to steady states with w/w_0 constant over time. Let $w_0=1$ so that w is the skilled relative to the unskilled wage. Dropping references to time t , and recalling the dependence of π on α and β (5.2.1)-(5.2.5) then become

$$(5.2.6) \quad C = L + (1-\beta) \quad C_0 = 1 - \phi_0 L \quad C_1 = w(\beta) - \phi_1 L$$

$$(5.2.7) \quad W = U(C) - \alpha + r \left[(1-\pi(\alpha, \beta))U(C_0) + \pi(\alpha, \beta)U(C_1) \right]$$

$$(5.2.8) \quad 1+n \leq (1-\pi(\alpha, \beta))\phi_0 + \pi(\alpha, \beta)\phi_1$$

Formally, the problems under consideration are as follows. If the government controls students' effort (as well as the repayment terms), then the individual faces the government-specified parameters (ϕ_0, ϕ_1, α) and chooses the expected utility-maximizing level of education. Thus, he solves

$$(5.2.9) \quad \underset{\beta}{\text{Max}} \quad (5.2.7) \quad \text{s.t.} \quad (5.2.6)$$

The solution is the function $\tilde{\beta}(\phi_0, \phi_1, \alpha)$. The government takes as given this behavior when it specifies the level of effort and repayment terms. The government's objective is to maximize social welfare. If all individuals are identical this implies

maximizing individual welfare. The government therefore solves

$$(5.2.10) \quad \text{Max}_{\phi_0, \phi_1, \alpha} \quad (5.2.7)$$

$$\text{s.t.} \quad \beta = \tilde{\beta}(\phi_0, \phi_1, \alpha) \quad (5.2.6) \text{ and } (5.2.8)$$

If the individual controls effort, he solves

$$(5.2.11) \quad \text{Max}_{\alpha, \beta} \quad (5.2.7) \quad \text{s.t.} \quad (5.2.6)$$

to generate the optimal levels of effort $\bar{\alpha}(\phi_0, \phi_1)$ and education $\bar{\beta}(\phi_0, \phi_1)$. The government takes as given this behavior in its choice of the repayment terms, solving

$$(5.2.12) \quad \text{Max}_{\phi_0, \phi_1} \quad (5.2.7)$$

$$\text{s.t.} \quad \alpha = \bar{\alpha}(\phi_0, \phi_1) \quad \beta = \bar{\beta}(\phi_0, \phi_1)$$

$$\text{s.t.} \quad (5.2.6) \text{ and } (5.2.8)$$

Section 5.3 considers the problems in (5.2.9) and (5.2.10), and section 5.4 considers the problems in (5.2.11) and (5.2.12).⁸

5.3 OPTIMAL SKILL, EFFORT AND INSURANCE WHEN THE GOVERNMENT CONTROLS STUDENTS' EFFORT

If $\tilde{\beta}$ is an internal solution, then it satisfies the first-order condition

$$(5.3.1) \quad U'(\tilde{C}) = r\pi(\alpha, \tilde{\beta})U'(\tilde{C}_1)w'(\tilde{\beta}) + r\pi^\beta(\alpha, \tilde{\beta})\Delta\tilde{U}$$

where $\tilde{C} = L + 1 - \tilde{\beta}$, $\tilde{C}_1 = w(\tilde{\beta}) - \phi_1 L$ and $\Delta U = U(C_1) - U(C_0)$. As is well-known, the individual chooses the level of education to equate the marginal utility from foregone earnings during the study period with the expected net marginal utility from education during the full-time work period. In this model the latter consists of the benefit from expected higher earnings (measured by the first right-hand side term), and the cost⁹ from a lower probability of graduating (measured by the second right-hand side term).

(5.3.1) implicitly defines $\tilde{\beta}(\phi_0, \phi_1, \alpha)$. Proposition 1 describes the behavior of $\tilde{\beta}$. It states that an increase in the level of effort raises the optimal level of education, and an increase in the per dollar repayment of an (un)skilled worker has a (non-)positive effect on the level of education:

Proposition 1

$$\frac{\partial \tilde{\beta}}{\partial \alpha} > 0 \quad \frac{\partial \tilde{\beta}}{\partial \phi_0} \leq 0 \quad = \quad \text{iff} \quad \pi^\beta = 0 \quad \frac{\partial \tilde{\beta}}{\partial \phi_1} > 0$$

Proof: All proofs are stated in section 5.6.

An increase in effort raises the probability of graduating, and hence the marginal benefit from education (the first right-hand side term in (5.3.1)). This induces the individual to study longer. Two effects operate when ϕ_0 and ϕ_1 are changed, working by way of the right-hand side terms in (5.3.1). The first effect is caused by a diminishing marginal utility from income (Assumption 1). An increase in ϕ_1 raises the expected marginal utility from graduating, inducing the individual to choose a longer education. On the other hand, the level of ϕ_0 does not affect the marginal return from graduating and so does not affect the level of education.¹⁰

The second effect works as follows. A fall in ϕ_0 or an increase in ϕ_1 reduces ΔU and hence $|r\pi^\beta \Delta U|$. As explained earlier, this term measures one of the costs of raising the level of education. Since that cost falls, the individual chooses a longer education. If $\pi^\beta = 0$ then there is no such effect, in which case changes in ϕ_0 have no impact on the level of education.

Given this behavior of the individual and given the budget constraint, the government chooses effort and the repayment terms to maximize social welfare (problem (5.2.10)). Proposition 2 shows that the government should choose the repayment terms so as to reduce the net wage differential

between skilled and unskilled workers. This involves demanding higher repayments from skilled than from unskilled workers (part (a)).¹¹ Further, unless the level of education has no impact on the probability of graduating, the government should leave some wage differential (part (b)).

Proposition 2

Let effort be controlled by the government. Let $(\hat{\phi}_0, \hat{\phi}_1, \hat{\alpha}, \hat{\beta})$ with $\hat{\beta} = \tilde{\beta}(\hat{\phi}_0, \hat{\phi}_1, \hat{\alpha})$ denote the optimum, and define $\hat{C}_0 = w_0 - \hat{\phi}_0 L$ and $\hat{C}_1 = w(\hat{\beta}) - \hat{\phi}_1 L$. Then

$$(a) \quad \hat{\phi}_0 < \hat{\phi}_1$$

$$(b) \quad \hat{C}_0 \leq \hat{C}_1 = \text{iff } \pi^{\beta} = 0$$

When the probability of graduating depends only on effort, and when the government fully controls effort, students' decisions do not generate externalities, so that it is optimal to free them from the risks associated with education. By guaranteeing a fixed return, the government provides students with full insurance.

If the government balances its budget in each period, the result implies that graduates pay back more than the amount they borrowed ($\hat{\phi}_1 > 1$). Further, if the population growth rate is zero (or very low), then balancing of the budget also implies that failures pay back less than the amount borrowed ($\hat{\phi}_0 < 1$). This property of the repayment scheme is independent

of the fact that the loan is fixed. When the population growth rate is high, graduates again repay more than the amount borrowed, but it is not clear how much failures repay in relation to the loan. Given that the loan is fixed, and that the amount of skill (and hence income) that can be accumulated by an individual is limited, and given that it is optimal that, after repaying their loans, graduates earn more than failures, a sufficiently high growth rate cannot be supported by graduates alone. Thus, failures would also have to repay more than they borrowed. However, this need not hold if the loan is endogenous. If the optimal size of the loan falls with an increase in the growth rate, then the income of graduates may be sufficient to support new generations of students.

The optimality of full insurance, when effort is controlled by the government, is suggested by the principal-agent model. However, a complete analogy with the principal-agent model would fix the duration (namely β) of the principal-agent relationship. In this model the length of education is controlled by the agent. Moreover, if it affects the probability of graduating, then the full insurance result no longer holds. An increase in the education level generates a negative externality (by reducing the expected repayment to the government), so that the education chosen by the student is longer than is socially optimal. By exposing the student to some of the associated risks he is induced to choose a shorter

education.¹²

Proposition 2 is also reminiscent of Eaton and Rosen's results. They show that it is optimal to use the tax system (a combination of a wage and a lumpsum tax) to insure against low future incomes. In the present model the loan scheme can be interpreted as a scheme of subsidies (L) and taxes ($\phi_0 L$ and $\phi_1 L$) the latter being used to provide insurance. A difference between the models is that these taxes can also be used as an incentive mechanism, influencing the individual's choices in education.

The government may be interested in how it should adjust the effort and repayments in response to changes in the population growth rate, and how this affects the optimal level of skill in the economy. This is addressed in Proposition 3 and Corollary 1 for a model in which the level of education does not affect the probability of graduating. It is found that an increase in the population growth rate raises the level of the repayments of all full-time workers, raises the level of student effort, and raises the level of education:

Proposition 3

Let effort be controlled by the government, and let $\pi^\beta = 0$. Then

$$\frac{d\hat{\phi}_i}{dn} > 0 \quad i=0,1 \quad \frac{d\hat{\alpha}}{dn} > 0$$

Corollary 1

Let effort be controlled by the government, and let $\pi^\beta = 0$. Then

$$\frac{d\hat{\beta}}{dn} > 0$$

In order to sustain the transfer of L per student to a larger student population, full-time workers are asked to make higher repayments. This is achieved by an increase in the repayments, and by an increase in effort, which (given $\hat{\phi}_0 < \hat{\phi}_1$) raises the per capita expected repayment and hence the total actual repayment. The increase in effort and the graduate repayment induces individuals to choose a longer education. Given that both effort and the level of education increase, a society with a higher population growth rate should exhibit a higher level of skill.

These results are limited by the assumption that the size of the loan is fixed. If the loan were variable, it appears likely that an increase in the population growth rate would lead the government to reduce the funding of individual students, as well as increase the repayments and level of effort in order to collect more funds; but, given a smaller loan, students are likely to choose a shorter education. This effect must be weighed against the effects of higher repayments and effort.

The present formulation may be relevant in the following

situation. Suppose that part-time work earns a very low wage, so that even if students work, they need at least an amount L in additional funding. Unless the alternative of no education is optimal, this places a bound on the downward adjustment in the loan. In such a case, a higher population growth rate causes little adjustment in the loan, and the above results apply.

5.4 OPTIMAL SKILL, EFFORT AND INSURANCE WHEN STUDENTS CONTROL EFFORT

This section repeats the analysis of section 5.3 for the case which allows students to effectively control the level of effort. Each individual chooses the level of effort and education to maximize expected utility (problem (5.2.11)). If $(\bar{\alpha}, \bar{\beta})$ is an interior solution, it satisfies the conditions

$$(5.4.1) \quad 1 = r\pi^{\alpha}(\bar{\alpha}, \bar{\beta}) \Delta \bar{U}$$

$$(5.4.2) \quad U'(\bar{C}) = r\pi(\bar{\alpha}, \bar{\beta}) U'(\bar{C}_1) w'(\bar{\beta}) + r\pi^{\beta}(\bar{\alpha}, \bar{\beta}) \Delta \bar{U}$$

where $\bar{C} = L + 1 - \bar{\beta}$, $\bar{C}_1 = w(\bar{\beta}) - \phi_1 L$ and $\Delta \bar{U} = U(\bar{C}_1) - U(\bar{C}_0)$. The interpretation of (5.4.2) is as before (see (5.3.1)). (5.4.1) states that at the optimal effort, the utility cost of an extra hour of study equals the discounted benefit from higher expected earnings. It implies that a positive effort requires

that skilled workers have a higher net income than unskilled workers. This is well-known from the principal-agent literature which suggests that students must be exposed to some of the risk associated with education, as an incentive to study.

(5.4.1) and (5.4.2) implicitly define $\bar{\alpha}(\phi_0, \phi_1)$ and $\bar{\beta}(\phi_0, \phi_1)$. The behavior of these functions is described in Proposition 4. If changes in effort affect the probability of graduating strongly (weakly) relative to changes in education, then an increase in insurance (an increase in ϕ_1 or a decrease in ϕ_0) lowers the level of effort, and lowers (raises) the level of education.

Proposition 4

Let effort be controlled by students. If $|\pi^\alpha/\pi^\beta|$ is large (small), then

$$\frac{\partial \bar{\alpha}}{\partial \phi_0} > 0, \quad \frac{\partial \bar{\alpha}}{\partial \phi_1} < 0, \quad \frac{\partial \bar{\beta}}{\partial \phi_0} > 0 \text{ } (<0), \quad \frac{\partial \bar{\beta}}{\partial \phi_1} < 0 \text{ } (>0)$$

If the burden of loan repayments shifts towards unskilled workers, then graduation becomes a more favorable outcome for the student, inducing him to study harder. Changes in ϕ_0 and ϕ_1 affect the level of education by way of the right-hand side terms in (5.4.1) and (5.4.2). An increase in insurance coverage reduces the cost of failure (the last term in

(5.4.2)), inducing an increase in the level of education. This effect also arises when the government controls effort. When the government does not control effort there is a further effect, operating via changes in the level of effort. By inducing a fall in effort, an increase in insurance lowers the probability of graduating and hence the benefit from education (the first right-hand side term in (5.4.2)), which makes a shorter education optimal. Ultimately, the effect depends on the size of π^α relative to π^β . If π^α is relatively large then additional insurance reduces the level of education. This result is opposite to the result in Proposition 1. Thus, when effort has a relatively large effect on the chances of graduating, the effect of insurance on the choice of education depends significantly on whether the government is able to control effort.

It was noted earlier that to induce a positive effort by students, the government should stop short of fully insuring them ($C_0^* < C_1^*$). A question is whether it should provide any insurance at all. Proposition 5 shows that as long as individuals are risk-averse some insurance is always optimal:

Proposition 5

Let effort be controlled by students. Let $(\phi_0^*, \phi_1^*, \alpha^*, \beta^*)$ with $\alpha^* = \alpha(\phi_0^*, \phi_1^*)$ and $\beta^* = \beta(\phi_0^*, \phi_1^*)$ denote the optimum, and define $C_0^* = w_0 - \phi_0^* L$ and $C_1^* = w(\beta^*) - \phi_1^* L$. Then $\phi_0^* < \phi_1^*$.

The principal-agent literature shows that if education is exogenous, then some insurance is optimal even if it induces a fall in effort. The endogeneity of education alters the optimal amount of insurance. Section 5.3 explains that with $\pi^\beta < 0$ the government would like to see a shorter education than chosen by the student, because education exerts a negative externality. Proposition 4 implies that if $|\pi^\alpha/\pi^\beta|$ is large (small) a shorter education can be achieved by an upward (downward) adjustment in the amount of insurance that is optimal if education does not generate such an externality.

The discussion of Proposition 2 regarding the relationship between the repayment and the amount borrowed (that is, whether $\phi_1^* > 1$ or $\phi_1^* < 1$) also applies here.

The effect of changes in the population growth rate when the education level has a negligible influence on the graduation rate, and the degree of risk-aversion is relatively small, is given in Proposition 6 and Corollary 2. It is shown that an increase in the population growth rate lowers (raises) the per dollar repayments of (un)skilled workers, and raises the levels of education and effort:

Proposition 6

Let effort be controlled by students. Let $\pi^\beta = 0$, and let $-U''/U'$ be sufficiently small. Then

$$\frac{d\phi_0^*}{dn} > 0 \quad \frac{d\phi_1^*}{dn} < 0$$

Corollary 2

Let effort be controlled by students. Let $\pi^\beta = 0$, and let $-U''/U'$ be sufficiently small. Then

$$\frac{d\alpha^*}{dn} > 0 \quad \frac{d\beta^*}{dn} > 0$$

An increase in the number of students per (full-time) worker requires larger repayments per worker. Given $\phi_0^* < \phi_1^*$ an increase in effort is one way of achieving this, since it increases the size of the skilled labor force. The government, however, cannot directly enforce a higher effort. As long as students are not too risk-averse, the government, as an incentive to raise effort, creates a larger net wage differential by shifting the burden of repayments towards unskilled workers. Moreover, such a change in the terms of repayment induces a longer education, which further enhances the government's ability to fund education.

5.5 CONCLUSION

This chapter extends the theory of education by modelling the problem of moral hazard, and generalizes

principal-agent theory by allowing the agent to choose the length of the principal-agent relationship.

A number of issues have been investigated. The first is the reaction of students to the government's provision of insurance against low future incomes (Propositions 1 and 4). It is found that with regard to the choice of education, students may react in opposite ways depending on whether they or the government control(s) the level of student effort. If the government controls effort, then an increase in the extent of insurance induces a higher level of education, because it raises the marginal utility from education and lowers the marginal cost of failure. If students control effort, and if effort rather than education levels significantly affect the chances of graduating, then an increase in insurance has the opposite effect on the level of education. The reason is that better coverage reduces the incentive to study. If this causes a relatively large fall in the probability of graduating, then students choose a shorter education.

A second issue is the degree of insurance that is socially optimal (Propositions 2 and 5). Existing principal-agent results imply that, for a given level of education, if the government controls students' effort, then it is socially optimal to fully insure future income, while partial insurance is optimal if students control effort. It is shown that these results extend to the case where the level of

education is variable as long as it does not affect the probability of graduating. However, if a longer education reduces the probability of graduating, then full insurance is suboptimal even if the government controls effort. The reason is that endogeneity of education essentially creates a second moral hazard problem. The government should use the loan scheme to induce a shorter education. If the government is able to control effort this can be achieved by a reduction in insurance. If the government is unable to control effort then the socially optimal course depends on whether the probability of graduating is affected more strongly by changes in effort or changes in education levels. If changes in effort (education) are the dominant influence, then an increase (reduction) in insurance is optimal.

If the government must implement a self-financing loan scheme, whereby workers' loan repayments completely finance new student loans, then it may be of interest to know how changes in the population growth rate affect the nature of the optimal loan scheme, and thereby the nature of education, and optimal levels of skill. This is described in Propositions 3 and 6 and Corollaries 1 and 2 for a model in which the probability of graduating depends only on the level of effort. It is shown that if the government controls effort, it will fund additional loans by raising the level of effort and repayments regardless of whether the student graduates. If the government is unable

to control effort, it will reduce (increase) the repayment of (un)skilled workers to motivate students. Whether or not the government controls effort, students choose a higher level of education. Hence, the model suggests that higher population growth rates are associated with higher levels of skill, and, if moral hazard is present, it suggests that higher population growth rates are also associated with greater exposure to risk. These comparative results are dependent on the assumption that the loan is fixed. It is conjectured that the level of skill may (although need not) decrease in response to higher population growth.

5.6 APPENDIX

Proof of Proposition 1

Let $\tilde{W}=W(\tilde{\beta})$. Total differentiation of (5.3.1) gives

$$\tilde{W}^{\beta\beta} = U''(\tilde{C}) + r\tilde{\pi}U''(\tilde{C}_1)[w'(\tilde{\beta})]^2 + r\tilde{\pi}U'(\tilde{C}_1)w''(\tilde{\beta})$$

$$\tilde{W}^{\beta\alpha} = r\tilde{\pi}^\alpha U'(\tilde{C}_1)w'(\tilde{\beta})$$

$$\tilde{W}^{\beta\phi_0} = r\tilde{\pi}^\beta U'(\tilde{C}_0)L$$

$$\tilde{W}^{\beta\phi_1} = -r\tilde{\pi}U''(\tilde{C}_1)w'(\tilde{\beta})L - r\tilde{\pi}^\beta U'(\tilde{C}_1)L$$

The result follows from Cramer's rule. ■

Proof of Proposition 2

Let \hat{L} be the Lagrangian of the problem in (5.2.10), and let τ be the multiplier associated with (5.2.8). If $(\hat{\phi}_0, \hat{\phi}_1, \hat{\alpha})$ is an interior solution, then it satisfies

$$(5.6.1) \quad \hat{L}^{\phi_i} = \tilde{W}^\beta \frac{\partial \tilde{\beta}}{\partial \phi_i} - r\hat{\pi}_i U'(\hat{C}_i)L + \hat{\tau}\hat{\pi}_i + \hat{\tau}\hat{\pi}^\beta (\hat{\phi}_1 - \hat{\phi}_0) \frac{\partial \tilde{\beta}}{\partial \phi_i} = 0$$

$$i=0,1 \quad \pi_i = \begin{cases} 1-\pi & \text{if } i=0 \\ \pi & \text{if } i=1 \end{cases}$$

where \tilde{W} and $\tilde{\beta}$ are evaluated at the optimum.

$$\Rightarrow r_{\pi_i}^{\hat{\pi}} U'(\hat{C}_i)_L = \hat{\tau} \left[\hat{\pi}_i + \frac{\hat{\pi}^\beta}{\hat{\pi}} (\hat{\phi}_1 - \hat{\phi}_0) \frac{\partial \tilde{\beta}}{\partial \phi_i} \right] \quad i=0,1 \quad \text{since } \tilde{W}^\beta=0$$

which implies

$$(5.6.2) \quad rU'(\hat{C}_i)_L = \hat{\tau} \left[1 + \frac{\hat{\pi}^\beta}{\hat{\pi}} (\hat{\phi}_1 - \hat{\phi}_0) \frac{\partial \tilde{\beta}}{\partial \phi_i} \right] \quad i=0,1$$

(a) Let $\hat{\phi}_0 \geq \hat{\phi}_1$. Then $\hat{C}_0 < \hat{C}_1$ and

$$\frac{\hat{\pi}^\beta}{1-\hat{\pi}} \left[\hat{\phi}_1 - \hat{\phi}_0 \right] \frac{\partial \tilde{\beta}}{\partial \phi_0} \leq \frac{\hat{\pi}^\beta}{\hat{\pi}} \left[\hat{\phi}_1 - \hat{\phi}_0 \right] \frac{\partial \tilde{\beta}}{\partial \phi_1}$$

using Proposition 1 and $\pi^\beta < 0$.

$$\Rightarrow U'(\hat{C}_0) \leq U'(\hat{C}_1) \quad \text{using (5.6.2)}$$

which contradicts the fact that $\hat{C}_0 < \hat{C}_1$. Hence $\hat{\phi}_0 < \hat{\phi}_1$.

(b) Let $\hat{C}_0 \geq \hat{C}_1$ and $\pi^\beta < 0$. Then $U'(\hat{C}_0) \leq U'(\hat{C}_1)$ and

$$1 - \hat{\phi}_0 L \geq w(\hat{\beta}) - \hat{\phi}_1 L$$

$$\Rightarrow \hat{\phi}_0 < \hat{\phi}_1$$

$$\Rightarrow \frac{\hat{\pi}^\beta}{\hat{\pi}_0} \left[\hat{\phi}_1 - \hat{\phi}_0 \right] \frac{\partial \tilde{\beta}}{\partial \phi_0} \geq \frac{\hat{\pi}^\beta}{\hat{\pi}_1} \left[\hat{\phi}_1 - \hat{\phi}_0 \right] \frac{\partial \tilde{\beta}}{\partial \phi_1} \quad \text{using Proposition 1}$$

$$\Rightarrow U'(\hat{C}_0) \geq U'(\hat{C}_1) \quad \text{using (5.6.2)}$$

which gives a contradiction. Hence, if $\pi^\beta < 0$, then $\hat{C}_0 < \hat{C}_1$.

Let $\pi^\beta = 0$. Then from (5.6.2)

$$rU'(\hat{C}_i)L = \hat{\tau} \quad i=0,1$$

$$\Rightarrow \hat{C}_0 = \hat{C}_1$$

Proof of Proposition 3

Given $\pi^\beta = 0$, the first-order conditions of problem (5.2.10) are

$$(5.6.3) \quad \hat{L}^{\phi i} = -r\hat{\pi}_i U'(\hat{C}_i)L + \hat{\tau}\hat{\pi}_i = 0 \quad i=0,1$$

$$(5.6.4) \quad \hat{L}^\alpha = -1 + r\hat{\pi}^\alpha \Delta \hat{U} + \hat{\tau}\hat{\pi}^\alpha [\hat{\phi}_1 - \hat{\phi}_0] = 0$$

$$(5.6.5) \quad \hat{L}^\tau = -(1+n) + (1-\hat{\pi})\hat{\phi}_0 + \hat{\pi}\hat{\phi}_1 = 0$$

Total differentiation of (5.6.3)-(5.6.5) gives

$$(5.6.6) \quad \begin{bmatrix} \hat{L}^{\phi_0\phi_0} & 0 & 0 & \hat{L}^{\phi_0\tau} \\ 0 & \hat{L}^{\phi_1\phi_1} & \hat{L}^{\phi_1\alpha} & \hat{L}^{\phi_1\tau} \\ 0 & \hat{L}^{\alpha\phi_1} & \hat{L}^{\alpha\alpha} & \hat{L}^{\alpha\tau} \\ \hat{L}^{\tau\phi_0} & \hat{L}^{\tau\phi_1} & \hat{L}^{\tau\alpha} & 0 \end{bmatrix} \begin{bmatrix} d\hat{\phi}_0 \\ d\hat{\phi}_1 \\ d\hat{\alpha} \\ d\hat{\tau} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ dn \end{bmatrix}$$

where

$$\hat{L}^{\phi} \phi_0 = r(1-\hat{\pi})U''(\hat{C}_0)L^2$$

$$\hat{L}^{\phi} \phi^{\tau} = 1 - \hat{\pi}$$

$$\hat{L}^{\phi} \phi_1 = -r\hat{\pi}U''(\hat{C}_1)L \left[w'(\hat{\beta}) \frac{\partial \tilde{\beta}}{\partial \phi_1} - L \right]$$

$$\hat{L}^{\phi} \phi^{\alpha} = -r\hat{\pi}U''(\hat{C}_1)w'(\hat{\beta})L \frac{\partial \tilde{\beta}}{\partial \alpha}$$

$$\hat{L}^{\phi} \phi^{\tau} = \hat{\pi}$$

$$\hat{L}^{\alpha\alpha} = r\hat{\pi}^{\alpha}U'(\hat{C}_1)w'(\hat{\beta}) \frac{\partial \tilde{\beta}}{\partial \alpha} + \hat{\tau}\hat{\pi}^{\alpha\alpha} \left[\hat{\phi}_1 - \hat{\phi}_0 \right]$$

$$\hat{L}^{\alpha\tau} = \hat{\pi}^{\alpha} \left[\hat{\phi}_1 - \hat{\phi}_0 \right]$$

where $\tilde{\beta}$ is evaluated at the optimum. The result follows from Cramer's rule, the first and second-order conditions and Assumptions 1-3. ■

Proof of Corollary 1

$\hat{\beta} = \tilde{\beta}(\hat{\phi}_0(n), \hat{\phi}_1(n), \hat{\alpha}(n))$ so that

$$\frac{d\hat{\beta}}{dn} = \frac{\partial \tilde{\beta}}{\partial \phi_0} \frac{d\hat{\phi}_0}{dn} + \frac{\partial \tilde{\beta}}{\partial \phi_1} \frac{d\hat{\phi}_1}{dn} + \frac{\partial \tilde{\beta}}{\partial \alpha} \frac{d\hat{\alpha}}{dn} > 0 \quad \text{by Propositions 1 and 3.}$$

Proof of Proposition 4

Let $\bar{W} = W(\bar{\alpha}, \bar{\beta})$. Total differentiation of (5.4.1)-(5.4.2) gives

$$\begin{bmatrix} \bar{W}^{\alpha\alpha} & \bar{W}^{\alpha\beta} \\ \bar{W}^{\beta\alpha} & \bar{W}^{\beta\beta} \end{bmatrix} \begin{bmatrix} d\bar{\alpha} \\ d\bar{\beta} \end{bmatrix} = \begin{bmatrix} -\bar{W}^{\alpha\phi_0} & -\bar{W}^{\alpha\phi_1} \\ -\bar{W}^{\beta\phi_0} & -\bar{W}^{\beta\phi_1} \end{bmatrix} \begin{bmatrix} d\phi_0 \\ d\phi_1 \end{bmatrix}$$

where

$$\bar{W}^{\alpha\alpha} = r\pi^{\alpha\alpha} \Delta \bar{U}$$

$$\bar{W}^{\alpha\beta} = r\pi^{\alpha} U'(\bar{C}_1) W'(\bar{\beta})$$

$$\bar{W}^{\alpha\phi_0} = r\pi^{\alpha} U'(\bar{C}_0) L$$

$$\bar{W}^{\alpha\phi_1} = -r\pi^{\alpha} U'(\bar{C}_1) L$$

$$\bar{W}^{\beta\beta} = U''(\bar{C}) + r\pi U''(\bar{C}_1) W'(\bar{\beta})^2 + r\pi U^{\alpha}(\bar{C}_1) W''(\bar{\beta}) + r\pi^{\beta\beta} \Delta \bar{U}$$

$$+ 2r\pi^{\beta} U'(\bar{C}_1) W'(\bar{\beta})$$

$$\bar{W}^{\beta\phi_0} = r\pi^{\beta} U'(\bar{C}_0) L$$

$$\bar{W}^{\beta\phi_1} = -r\bar{\pi}U''(\bar{C}_1)W'(\bar{\beta})L - r\bar{\pi}^{\beta}U'(\bar{C}_1)L$$

The result follows from Cramer's rule, the first and second-order conditions, and Assumptions 1-3. ■

Proof of Proposition 5

Let \bar{L}^* be the Lagrangian of the problem in (5.2.12). If (ϕ_0^*, ϕ_1^*) is an interior solution, then it satisfies

$$(5.6.7) \quad \bar{L}^{\phi_i} = \bar{W}^{\beta} \frac{\partial \bar{\beta}}{\partial \phi_i} + \bar{W}^{\alpha} \frac{\partial \bar{\alpha}}{\partial \phi_i} - r\pi^* U'_i(\bar{C}_i)L + \tau\pi^{*\alpha} [\phi_1^* - \phi_0^*] \frac{\partial \bar{\alpha}}{\partial \phi_i} \\ + \tau\pi^{*\beta} [\phi_1^* - \phi_0^*] \frac{\partial \bar{\beta}}{\partial \phi_i} + \tau\pi_i^{**} = 0 \quad i=0,1$$

where \bar{W} , $\bar{\alpha}$ and $\bar{\beta}$ are evaluated at the optimum.

$$\Rightarrow -rU'(\bar{C}_i)L + \tau [\phi_1^* - \phi_0^*] \left[\frac{\pi^{\alpha}}{\pi_i^*} \frac{\partial \bar{\alpha}}{\partial \phi_i} + \frac{\pi^{\beta}}{\pi_i^*} \frac{\partial \bar{\beta}}{\partial \phi_i} \right] + \tau = 0 \quad i=0,$$

using $\bar{W}^{\alpha} = \bar{W}^{\beta} = 0$.

$$\Rightarrow \bar{L}^{\phi_0} - \bar{L}^{\phi_1} = -rL \left[U'(\bar{C}_0) - U'(\bar{C}_1) \right] \\ + \tau [\phi_1^* - \phi_0^*] \left[\frac{\pi^{\alpha}}{1-\pi} \frac{\partial \bar{\alpha}}{\partial \phi_0} - \frac{\pi^{\alpha}}{\pi} \frac{\partial \bar{\alpha}}{\partial \phi_1} + \frac{\pi^{\beta}}{1-\pi} \frac{\partial \bar{\beta}}{\partial \phi_0} - \frac{\pi^{\beta}}{\pi} \frac{\partial \bar{\beta}}{\partial \phi_1} \right] = 0$$

$$\Rightarrow \phi_1^* - \phi_0^* = \frac{rL[U'(\check{C}_0^*) - U'(\check{C}_1^*)]}{\tau \left[\frac{\pi^{\alpha}}{1-\pi^*} \frac{\partial \bar{\alpha}}{\partial \phi_0} - \frac{\pi^{\alpha}}{\pi^*} \frac{\partial \bar{\alpha}}{\partial \phi_1} + \frac{\pi^{\beta}}{1-\pi^*} \frac{\partial \bar{\beta}}{\partial \phi_0} - \frac{\pi^{\beta}}{\pi^*} \frac{\partial \bar{\beta}}{\partial \phi_1} \right]}$$

The numerator is positive if (5.4.1) holds. The first two terms in the denominator are positive by Proposition 4.

Let $|\pi^{\alpha}/\pi^{\beta}|$ be relatively small. Then

$$\frac{\partial \bar{\beta}}{\partial \phi_0} < 0 \quad \text{and} \quad \frac{\partial \bar{\beta}}{\partial \phi_1} > 0$$

so that, given $\pi^{\beta} < 0$, the last two terms in the denominator are also positive. Let $|\pi^{\alpha}/\pi^{\beta}|$ be relatively large. Then the last two terms in the denominator are negative, but outweighed by the first two terms. Hence $\phi_1^* - \phi_0^* > 0$. ■

Proof of Proposition 6

Given $\pi^{\beta} = 0$, the first-order conditions of problem (5.2.12) are

$$(5.6.8) \quad L^{\phi_i} = -r\pi_i^* U'(\check{C}_i^*)L + \frac{\pi^{\alpha}}{\tau} [\phi_1^* - \phi_0^*] \frac{\partial \bar{\alpha}}{\partial \phi_i} + \pi_i^* = 0 \quad i=0,1$$

$$(5.6.9) \quad L^{\tau} = -(1+n) + (1-\pi^*)\phi_0^* + \pi^*\phi_1^* = 0$$

where $\bar{\alpha}$ is evaluated at the optimum.

Total differentiation of (5.6.8)-(5.6.9) gives

$$\begin{bmatrix} \overset{*}{L}^{\phi\phi}_0 & \overset{*}{L}^{\phi\phi}_1 & \overset{*}{L}^{\phi\tau} \\ \overset{*}{L}^{\phi 1}_0 & \overset{*}{L}^{\phi 1}_1 & \overset{*}{L}^{\phi 1\tau} \\ \overset{*}{L}^{\tau\phi}_0 & \overset{*}{L}^{\tau\phi}_1 & 0 \end{bmatrix} \begin{bmatrix} d\overset{*}{\phi}_0 \\ d\overset{*}{\phi}_1 \\ d\tau \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ dn \end{bmatrix}$$

where

$$\begin{aligned} \overset{*}{L}^{\phi i\phi i} &= r\pi_i^* U''(C_i^*) L^2 - r\pi_i^{\alpha} U'(\overset{*}{C}_i) L \frac{\partial \bar{\alpha}}{\partial \phi_i} + 2\pi_i^{\alpha} \frac{\partial \bar{\alpha}}{\partial \phi_i} \\ &\quad + \pi^{\alpha\alpha} \left[\overset{*}{\phi}_1 - \overset{*}{\phi}_0 \right] \left[\frac{\partial \bar{\alpha}}{\partial \phi_i} \right]^2 + \pi^{\alpha\alpha} \left[\overset{*}{\phi}_1 - \overset{*}{\phi}_0 \right] \frac{\partial^2 \bar{\alpha}}{\partial \phi_i^2} \quad i=0,1 \\ \overset{*}{L}^{\phi\phi}_1 &= \left[r\pi^{\alpha} U'(\overset{*}{C}_0) L - \pi^{\alpha\alpha} \right] \frac{\partial \bar{\alpha}}{\partial \phi_1} + \pi^{\alpha\alpha} \frac{\partial \bar{\alpha}}{\partial \phi_0} + \pi^{\alpha\alpha} \left[\overset{*}{\phi}_1 - \overset{*}{\phi}_0 \right] \frac{\partial \bar{\alpha}}{\partial \phi_0} \frac{\partial \bar{\alpha}}{\partial \phi_1} \\ &\quad + \pi^{\alpha\alpha} \left[\overset{*}{\phi}_1 - \overset{*}{\phi}_0 \right] \frac{\partial^2 \bar{\alpha}}{\partial \phi_0 \partial \phi_1} \\ \overset{*}{L}^{\phi\tau} &= 1 - \pi + \pi^{\alpha} \left[\overset{*}{\phi}_1 - \overset{*}{\phi}_0 \right] \frac{\partial \bar{\alpha}}{\partial \phi_0} \\ \overset{*}{L}^{1\tau} &= \pi + \pi^{\alpha} \left[\overset{*}{\phi}_1 - \overset{*}{\phi}_0 \right] \frac{\partial \bar{\alpha}}{\partial \phi_1} \end{aligned}$$

The result follows from Cramer's rule, the first and second-order conditions, Assumptions 1-3, and the assumption

that the degree of risk-aversion is sufficiently small. ■

Proof of Corollary 2

$\alpha^* = \bar{\alpha}(\phi_0^*(n), \phi_1^*(n))$ and $\beta^* = \bar{\beta}(\phi_0^*(n), \phi_1^*(n))$, so that

$$\frac{d\alpha^*}{dn} = \frac{\partial \bar{\alpha}}{\partial \phi_0} \frac{d\phi_0^*}{dn} + \frac{\partial \bar{\alpha}}{\partial \phi_1} \frac{d\phi_1^*}{dn} > 0 \quad \text{by Propositions 4 and 6; and}$$

$$\frac{d\beta^*}{dn} = \frac{\partial \bar{\beta}}{\partial \phi_0} \frac{d\phi_0^*}{dn} + \frac{\partial \bar{\beta}}{\partial \phi_1} \frac{d\phi_1^*}{dn} > 0 \quad \text{by Propositions 4 and 6.} \quad \blacksquare$$

5.7 NOTES

1. In the present model students act as agents and the government acts as the principal.
2. The possibility of double moral hazard, with students choosing suboptimal levels of education and effort, is similar to the problems discussed in Chapter 3. See also note 12 below.
3. This model modifies and extends the model used by Eaton and Rosen (1980).
4. A change in the ratio of skilled to unskilled labor affects the marginal product of skilled relative to unskilled labor, and in a competitive world, affects the skilled relative to unskilled wage. This effect will be incorporated in Chapter 6.
5. The individual also has time for leisure. The allocation of time to leisure and productive activities is not considered.
6. Each period should be thought of as consisting of a certain number of days. Each day consists of an exogenous number of productive hours and leisure hours. β measures the number of productive hours in each day allocated to attending school. α measures the number of leisure hours allocated to study. During the remainder of the day the individual performs unskilled work.

7. This assumes a direct relationship between the time spent in the education system and the level of the associated degree.
8. The analysis does not consider the dynamic properties of the model.
9. This assumes $\Delta U > 0$. If $\Delta U < 0$ the term $r\pi^\beta \Delta U$ measures a benefit from education. The cost interpretation is given here and below, because it will be shown that $\Delta U < 0$ is suboptimal.
10. Having made the choice to have an education, (this is implicit in the assumption that $\hat{\beta}$ is an internal solution) the net income of an unskilled worker is not relevant in determining the optimal level of education.
11. If the taxation/subsidy interpretation is adopted this suggests that progressive taxation is optimal.
12. When $\pi^\beta < 0$ problems (5.2.10) and (5.2.12) resemble problems (3.3.6) and (3.3.7). The solution to (3.3.6) identified in Chapter 3 is full insurance of the return to the agent, inducing him to let the principal control β . That is not optimal in this case, because even with full insurance ($C_0 = C_1$) the expected utility of the individual is not independent of β .

CHAPTER 6

INCENTIVES IN A GENERAL EQUILIBRIUM EDUCATION MODEL

6.1 INTRODUCTION

The model of Chapter 5 is restricted by the assumption that foregone earnings are the only cost of education. For instance, there is no explicit modelling of the labor required to train students. Secondly, all individuals choose to educate themselves before entering the workforce. Thirdly, the effect of changes in the composition of the workforce on relative wages is ignored. As such, the model is not a general equilibrium model.

Manning (1975, 1976, 1985) describes a model in which skilled workers are diverted from the production of consumer goods in order to become educators. The cost of education to students is the cost of tuition to cover educators' salaries as well as the cost of students' foregone earnings. The model adds realism by allowing individuals to follow different career paths. Some individuals choose unskilled jobs, while others invest in education and, upon graduating, choose between jobs in the education or consumer goods sectors. Further, real wages are determined endogenously.

The relevance of the model to principal-agent theory is

as follows. The government (which acts as a principal) hires educators (who act as agents) to train students. The effort of the educators is productive in the sense that it raises the probability that students graduate. Since graduates are more productive (and hence consume more) than failures, each student has an interest in high effort on the part of his educator. But, if effort generates disutility, then educators have an incentive to shirk. That is, a moral hazard problem arises. The remedy comes from the principal-agent literature, namely that the contract, which the government offers to educators, should contain penalties for a low graduation rate.

The government's role in education is purely as an intermediary, channeling payment from students to educators. Equally well, each student can write a contract with the educator of his class. Tuition fees would then be dependent on whether the student graduates. The model assumes that all individuals are risk-neutral, so that students are not harmed by the fact that tuition fees are uncertain. (If students are risk-averse, they can each write a side contract with an insurance company.)

The model generalizes existing principal-agent theory by taking into account aspects of general equilibrium. In particular, the number of agents (the educators) in the present model is endogenously determined. Their opportunity cost, which is the skilled wage in the consumer good sector plus the

disutility from effort is also endogenous (that is, \bar{U} in constraint (1.2.1) in Chapter 1 is endogenous). These generalizations complicate a comparative analysis.

To simplify the analysis, issues of insurance, which are emphasized in Chapter 5, are ignored by assuming that educators are risk-neutral. This enables the government to write a contract, which completely eliminates moral hazard. At issue are the characterization of such a contract, and adjustments in the terms of the contract to changes in the environment.

The following section introduces moral hazard into Manning's model. Section 6.3 characterizes the optimal levels of skill and effort, and the contract which the planner would offer to educators, if the planner could observe their activities. Section 6.4 considers the case where the planner cannot observe those activities, describes a contract which implements the optimal level of effort, and derives some comparative steady state results.

6.2 A GENERAL EQUILIBRIUM EDUCATION MODEL

Consider the following planning model specified in the papers by Manning. At any point in time, a labor force L consists of skilled (S) and unskilled (U) workers; that is,

$$L = S + U$$

The labor force is engaged either in a consumer good sector or an education sector.

In the consumer good sector skilled (X) and unskilled (Y) workers produce a consumer good (C):

$$C = f(X, Y)$$

where f is a concave, constant returns to scale production function.^{1,2}

In the education sector educators (who are drawn from the skilled workforce and are denoted E) train students (who belong to the unskilled workforce and are denoted T) producing graduates (G) by way of a fixed coefficients production function. Specifically, if $s = T/E$ is an exogenously given student-staff ratio, and ζ is an exogenously given graduation rate, then

$$G = Es\zeta$$

Let n be the population growth rate, and let m be the death rate of skilled workers. Then, under balanced growth, a proportion $v = n + m$ of the total skilled workforce must be replaced by new graduates; that is,

$$(6.2.1) \quad Es\zeta = vS$$

Given the needs of the education sector, and given the requirement of balanced growth, the labor available for

consumer good production can be written in per capita terms as

$$(6.2.2) \quad x = \frac{X}{L} = \frac{S-E}{L} = \left(1 - \frac{V}{s\zeta}\right) \sigma \quad \sigma = \frac{S}{L}$$

$$(6.2.3) \quad y = \frac{Y}{L} = \frac{L-S-T}{L} = 1 - \left(1 + \frac{V}{\zeta}\right) \sigma$$

so that per capita consumption under balanced growth is

$$(6.2.4) \quad c = \frac{C}{L} = f\left(\left(1 - \frac{V}{s\zeta}\right) \sigma, 1 - \left(1 + \frac{V}{\zeta}\right) \sigma\right)$$

Assuming that consumption of the consumer good alone generates utility, a planner chooses the proportion of skilled labor which maximizes per capita consumption. Thus, the planner solves

$$\text{Max}_{\sigma} \quad c \quad \text{s.t.} \quad (6.2.4)$$

This generates the result that at a welfare maximum

$$(6.2.5) \quad \frac{f^X}{f^Y} = \frac{1 + \frac{V}{\zeta}}{1 - \frac{V}{s\zeta}}$$

In words, the rate at which skilled labor can be substituted for unskilled labor in the production of the consumer good equals the rate at which unskilled labor can be transformed into skilled labor, after netting out the labor requirements of

the education sector.

The model is now extended by making the graduation rate explicitly dependent on the quality of students, and effort of the educator. Suppose that there are many identical classes, each with one educator and s students. The outcome in each class is either a low graduation rate ζ_1 , or a high graduation rate ζ_2 . An educator can increase the probability π , that a high proportion of his class graduates, by exerting more effort. Let $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ denote an educator's effort. Assume that the educator's effort is subject to diminishing returns. Then $\pi = \pi(\alpha)$, with $\pi'(\cdot) > 0$ and $\pi''(\cdot) < 0$. Under these circumstances, if a class performs poorly (if ζ_1 is observed for that class), the cause may be a lack of talent or effort³ on the part of students, or shirking by the educator.

It is assumed that the probability function π is independent across classes, so that, given sufficiently many classes, the actual graduation rate ζ for the entire student population equals the expected class graduation rate; that is,

$$\zeta = (1-\pi)\zeta_1 + \pi\zeta_2$$

The balanced growth condition in (6.2.1) and the definitions of x and y in (6.2.2) and (6.2.3) therefore still apply. However, ζ is now determined endogenously.

The planner must choose the welfare-maximizing levels of skill and effort. If consumption of the good alone affects the

level of utility, then, since

$$f^{\alpha} = \frac{v\sigma\pi^{\alpha}\Delta\zeta}{\zeta^2} \left(\frac{1}{s} f^X + f^Y \right) > 0,$$

where $\Delta\zeta = \zeta_2 - \zeta_1$, the optimal choice of effort is the maximum level $\bar{\alpha}$. However, if effort involves a utility cost, then society will generally prefer an output level that is below the maximum level.

It is assumed that individuals have identical tastes and regard a unit of the good in consumption as good as a unit saved in effort. On the basis of this, the planner maximizes total output of the good net of the quantity of output, which would just compensate educators for their effort. Let A be the total effort by educators. Then given the constant marginal rate of substitution between the good and effort in consumption (and given suitable units of measuring utility), the planner maximizes $C-A$.

Although individuals have identical tastes, actual utility derived from consumption of the good or from effort need not be identical. For example, educators exert effort, so that their disutility from effort exceeds that of skilled workers in the consumer good sector. However, if an equilibrium is to prevail, educators must consume more of the good as compensation. More generally, equilibrium requires that, even if individuals have different careers, they must

expect the same lifetime utility level. The planner therefore maximizes representative utility, or

$$W \equiv \frac{C-A}{L}$$

From (6.2.1), under balanced growth the number of educators is $vS/s\zeta$, so that effort per head of the population⁴ is

$$\frac{A}{L} = \frac{v\sigma}{s\zeta}\alpha$$

and W becomes

$$(6.2.6) \quad W = c - \frac{v\sigma}{s\zeta}\alpha$$

The planner wants to be able to implement the optimal level of effort $\hat{\alpha}$, which is found (along with the optimal level of skill $\hat{\sigma}$) by solving

$$(6.2.7) \quad \underset{\sigma \quad \alpha}{\text{Max}} \quad c - \frac{v\sigma}{s\zeta}\alpha \quad \text{s.t.} \quad (6.2.4)$$

If the planner is able to observe the educator's effort, then he can offer each educator a forcing contract of the form described in the principal-agent literature (see, for instance, Harris and Raviv, 1979). Such a contract is described in section 6.3.

If effort is not observable, then the planner can only induce educators to choose the optimal level of effort. In the

principal-agent literature it has been shown that, if agents are risk-neutral, then the optimum can be implemented. This also holds for the present model. Section 6.4 derives the contract in terms of the optimal values which the planner wishes to implement. A characterization of the optimum is now given.

6.3 OPTIMAL LEVELS OF SKILL AND EFFORT

An interior solution $(\hat{\sigma}, \hat{\alpha})$ to (6.2.7) must satisfy

$$(6.3.1) \quad \left(1 - \frac{v}{s\hat{\zeta}}\right) \hat{f}^X - \left(1 + \frac{v}{\hat{\zeta}}\right) \hat{f}^Y - \frac{v}{s\hat{\zeta}} \hat{\alpha} = 0$$

$$(6.3.2) \quad \frac{v\hat{\sigma}\hat{\pi}^{\alpha}\Delta\hat{\zeta}}{\hat{\zeta}^2} \left(\frac{1}{s} \hat{f}^X + \hat{f}^Y\right) + \frac{v\hat{\sigma}\hat{\pi}^{\alpha}\Delta\hat{\zeta}}{s\hat{\zeta}^2} \hat{\alpha} - \frac{v}{s\hat{\zeta}} \hat{\sigma} = 0$$

Condition (6.3.1) extends the result in (6.2.5). The first two terms measure the extra consumption generated by a marginal increase in the level of per capita skill. The third term measures the loss in welfare from the effort put forth by the additional educators needed to produce the additional skill. At a welfare maximum the marginal benefit from the gain in (per capita) consumption equals the marginal cost from the increase in (per capita) effort.

Rearranging (6.3.1) gives an alternative interpretation, namely

$$(6.3.3) \quad \frac{\hat{f}^X - \frac{v_\alpha}{\hat{s}_\zeta} \frac{1}{(1 - \frac{v}{\hat{s}_\zeta})}}{\hat{f}^Y} = \frac{1 + \frac{v}{\hat{s}_\zeta}}{1 - \frac{v}{\hat{s}_\zeta}}$$

or, the marginal rate of substituting skilled for unskilled labor in the consumer good sector (holding welfare constant)⁵ equals the marginal rate of transforming unskilled into skilled labor.

In contrast to Manning's result in (6.2.5), (6.3.3) implies

$$(6.3.4) \quad \frac{\hat{f}^X}{\hat{f}^Y} > \frac{1 + \frac{v}{\hat{s}_\zeta}}{1 - \frac{v}{\hat{s}_\zeta}}$$

Intuitively, the cost of a skilled worker, in terms of the unskilled labor needed in its production (the right-hand side expression) is smaller than the amount of unskilled labor that can be saved by using one more skilled worker in the consumer good sector. The difference reflects the cost of the effort needed to produce the additional skilled worker.

Condition (6.3.4) may be illustrated diagrammatically using figure 2 in Manning (1985), which is reproduced in Figure 6.8.1. It shows society's input possibilities frontier, and isoquants of the consumer good production function. When

output is maximized (6.2.5) holds, and the corresponding isoquant is tangential to the input possibilities frontier (point a). (6.3.3) implies that at a welfare maximum, the relevant isoquant is steeper than the input possibilities frontier (see point b), so that (as suggested in section 6.2) output is not maximized. Further, given the same input possibilities, the levels of output and skill are lower than in Manning's model.

Condition (6.3.2) is explained as follows. The term $\hat{v}_{\sigma\pi}^{\alpha} \hat{\Delta\zeta}/\hat{\zeta}^2$ measures the effect of a marginal increase in effort on the number of students who can be released from the education sector, while maintaining a given level of skill. Given the fixed coefficients technology $(1/s)(\hat{v}_{\sigma\pi}^{\alpha} \hat{\Delta\zeta}/\hat{\zeta}^2)$ is the number of educators who are simultaneously released. Hence, the first term in (6.3.2) measures the benefit to society, in terms of additional consumption, from a marginal increase in effort. The second term measures the gain from effort saved by the release of educators. However, the increase in effort involves a direct cost - effort by existing educators increases (on a per capita basis), the cost of which is the third term in (6.3.2). Thus, at the optimal level of effort, the social marginal cost and benefit of effort are equal.

Rearranging (6.3.2) gives

$$(6.3.5) \quad \frac{\frac{\hat{\pi}^\alpha \Delta \hat{\zeta}}{\hat{\zeta}} \left(\frac{1}{s} \hat{f}^X + \hat{f}^Y \right)}{\frac{1}{s} \left(1 - \frac{\hat{\pi}^\alpha \Delta \hat{\zeta}}{\hat{\zeta}} \hat{\alpha} \right)} = 1$$

In words, the marginal rate of transforming effort into the consumer good by way of a release of labor from education to the consumer good sector⁶ equals the marginal rate of substitution between effort and the good in consumption.

The optimal values $\hat{\sigma}$ and $\hat{\alpha}$ determine the optimal competitive (real) skilled wage \hat{f}^X , and hence the amount of tuition charged to students, namely $1/s(\hat{f}^X + \hat{\alpha})$, which induces skilled workers to accept jobs as educators. The effort $\hat{\alpha}$ is implemented by way of a forcing contract such as described in Harris and Raviv (1979). Thus, if \hat{c} is the optimal payment per educator, the contract offered by the planner to each educator is

$$(6.3.6) \quad \begin{aligned} \hat{c} &= \hat{f}^X + \hat{\alpha} & \text{if } \alpha &= \hat{\alpha} \\ \hat{c} &= 0 & \text{if } \alpha &\neq \hat{\alpha} \end{aligned}$$

The parameters in the model are the technology in education (s, ζ_1, ζ_2) ⁷, and the rate at which skilled labor must be replaced by graduates to maintain balanced growth (v - referred to below as the replacement rate). The effects of changes in these parameters on the optimal levels of skill and

effort are derived in Appendix 1, and summarized in Table 1.

Table 1^{8,9}

	s	ζ_1	ζ_2	v
$\hat{\sigma}$	+	+	+	-
$\hat{\alpha}$	+	-	+	+

Table 1 shows that an increase in the student-staff ratio raises the optimal levels of skill and effort. Since the graduation rate does not depend on the student-staff ratio, an increase in this ratio means that each educator has become more productive. Skilled labor on average (combining educators and skilled workers in consumer good production) therefore has a higher marginal product, which raises the optimal level of skill.

Since effort has the properties of a public good, (within limits, new students in a class can benefit as well as the existing students from a given level of effort by the educator), an increase in the student-staff ratio makes effort more productive, and raises the optimal level of effort. This is the direct effect of an increase in s.

There is also an indirect effect, caused by the increase in the optimal level of skill. On the one hand, a greater level of skill lowers the marginal product of skilled labor f^x ; on the other hand, since more students are diverted to the

education sector, the marginal product of unskilled labor f^Y rises. Suppose that the second effect dominates. Then, the greater is s , the greater is the change in $f^X + sf^Y$. Now this term measures the output gained from releasing one educator and s students through an increase in effort. The larger this benefit from more effort, the higher the optimal level of effort. In this case, the indirect effect reinforces the direct effect. If instead a fall in f^X dominates the rise in f^Y , then an increase in s has a negative indirect effect on optimal effort. In either case, the second-order conditions require that the indirect effect be relatively small.

An increase in the graduation rates ζ_1 or ζ_2 raises the expected graduation rate of a given class, and the actual graduation rate for the total student population. This means that the production of skill requires a smaller quantity of resources (labor and effort) and therefore raises the optimal level of skill.

The higher the expected graduation rate, the cheaper the production of a given level of skill. If ζ_2 increases, the expected graduation rate increases more, the higher the level of effort. Hence, increases in ζ_2 make effort more productive. In contrast, a given increase in ζ_1 raises the expected graduation rate more, the lower the level of effort. This explains the optimality of higher effort when ζ_2 increases, and

lower effort when ζ_1 increases.

A higher replacement rate implies that more students and educators are needed in any given period to maintain a given level of per capita skill in future periods. By thus increasing the output loss as well as the total effort, the marginal cost of (per capita) skill rises. Since skill (embodied in educators) and effort are substitutes in the production of graduates, this suggests that more effort should be expanded in education.

6.4 FORCING CONTRACTS WHEN EFFORT IS UNOBSERVABLE

Suppose that effort is unobservable, so that the government cannot specify effort in a contract. The principal-agent literature has shown that, when the agent is risk-neutral, the optimal action can be implemented by means of suitably specified payments (for instance, Shavell, 1979b). Such a forcing contract is derived below for the present general equilibrium model. Since the contract is written exclusively in terms of the variables derived in section 6.3, the results in Table 1, and the properties of f and π are sufficient to derive some comparative steady state results about the nature of the contract.

As supposed in the discussion of section 6.3, students pay the planner tuition fees to finance the salaries of

educators. When effort is observable, the planner pays educators an amount just covering the wage they could have earned in the consumer good sector plus an amount to compensate for the disutility from effort. The payment is independent of the observed graduation rate. If the graduation rate is low, the planner realizes that the educator has drawn a class of low quality.

When effort is unobservable the planner motivates educators by paying a larger amount if a class has a high graduation rate, on the basis that a high graduation rate is evidence of high effort. Let c_1 and c_2 denote the payment to each educator in the event of a low and high graduation rate, respectively. Since the educator is risk-neutral the risk of receiving a low payment in the event that he draws a poor class does not generate a utility loss. However, to attract skilled workers into the education sector, they must be paid at least their opportunity cost. For example, if the planner wants to implement a level of skill $\hat{\sigma}$ and a level of effort $\hat{\alpha}$, then the contract (c_1, c_2) must satisfy

$$(6.4.1) \quad (1-\hat{\pi})c_1 + \hat{\pi}c_2 - \hat{\alpha} = \hat{f}^x$$

where the left-hand side measures an educator's expected utility level. This is the general equilibrium version of the participation constraint in the principal-agent model. It differs in that the opportunity cost of an educator is

determined endogenously (as described in section 6.3). If educators do not receive at least an expected wage covering the skilled wage plus compensation for effort, then skilled workers will take jobs in the consumer good sector until the marginal return to both types of skilled labor is equalized.

To ensure that an educator chooses the optimal level of effort $\hat{\alpha}$, the planner must make the contract compatible with the choices of the educator. The contract must satisfy the incentive-compatibility constraint

$$(6.4.2) \quad \hat{\pi}^{\alpha} (c_2 - c_1) - 1 = 0$$

Simultaneously solving (6.4.1) and (6.4.2) gives the contract (\hat{c}_1, \hat{c}_2) , where

$$(6.4.3) \quad \hat{c}_1 = \hat{f}^x + \hat{\alpha} - \frac{\hat{\pi}}{\hat{\pi}^{\alpha}} \quad \hat{c}_2 = \hat{f}^x + \hat{\alpha} + \frac{1 - \hat{\pi}}{\hat{\pi}^{\alpha}}$$

This differs from the payment \hat{c} , defined in (6.3.6), only in the third terms in (6.4.3) which quantify the incentives in the contract. Since the contracts (\hat{c}_1, \hat{c}_2) and $(\hat{c}, \hat{\alpha})$ both implement the optimal effort, the only difference between a world in which effort is observable and one in which effort is unobservable are the penalties $(\hat{c}_2 - \hat{c}_1) = 1/\hat{\pi}^{\alpha}$ imposed in the latter case. In this world, ex post, some educators are worse off than others, not because they shirked, but because they drew a poor class. However, penalties are necessary to prevent

shirking.

The planner changes the optimal penalty in response to changes in the environment (s, ζ_1, ζ_2) . The optimal adjustments are derived in Appendix 2, section 6.7, and are summarized in:

Table 2

	s	ζ_1	ζ_2	v
$\hat{c}_2 - \hat{c}_1$	+	-	+	+

An increase in the student-staff ratio gives each educator a larger class, so that the educator's effort becomes more important. The optimal penalty for (perceived) poor performance therefore increases.

If ζ_2 increases or ζ_1 decreases, the cost of a poor class performance increases. A larger penalty in the event of a low graduation rate is therefore optimal.

A higher replacement rate implies that more resources are needed to maintain the production of skill. Effort is one such resource and more can be obtained by higher penalties.

The planner must determine not only optimal contracts for educators, but also tuition fees to students. Although the wage of an individual educator is uncertain, the total wage bill faced by the planner is determinate, because there are many educators. Hence, the planner knows with certainty how much tuition to charge to students. The effect on tuition fees

of changes in the parameters depends on how those changes affect the optimal skilled wage \hat{f}^x , and the optimal effort $\hat{\alpha}$. Unfortunately, unambiguous results require more specific knowledge of the functions f and π . A definite result is available only for the effect of a change in the student-staff ratio. If this ratio increases, then the optimal skilled wage falls, and the share which each student must contribute to the compensation of educators also falls. Although the optimal effort of each educator increases and requires compensation, as long as the maximum effort is sufficiently small,¹¹ tuition fees on a per student basis fall.

6.5 CONCLUSION

In the context of a balanced growth model in which a planner determines the allocation of labor to the education and consumer good sectors, and the effort desired from educators, conditions are derived characterizing a welfare-maximum. It is shown that, at a welfare-maximum, the marginal rate of substituting skilled for unskilled labor in the production of the good, equals the marginal rate of transforming unskilled into skilled labor through education. Also, the marginal rate of transforming effort into the good by a release of labor from education equals the marginal rate of substituting effort for the good in consumption.

Since an educator's effort has benefits (namely, an increase in the productivity of his students) which are not privately captured, the planner may face moral hazard if he is unable to observe the educator's activities. A penalty on the educator in the event of a low graduation rate in the educator's class can alleviate the moral hazard. If the educator is risk-neutral, a penalty can completely eliminate moral hazard. This is familiar from the principal-agent literature. However, in the present model, the planner must calculate the optimal penalty in a world where the number of educators, as well as their opportunity cost are endogenously determined.

The planner solves for the optimal penalty in two stages, first calculating the optimal levels of skill and effort, and then choosing performance-dependent payments. The planner must make sure that the payments equate the expected return to skilled labor across sectors. This is a general equilibrium version of the participation constraint in the conventional principal-agent model. At the same time, the penalty implicit in the payments must induce educators to choose the optimal level of effort.

The comparative steady state results show that the optimal penalty is larger, the higher the student-staff ratio, the higher the graduation rate of classes which perform well, the lower the graduation rate of classes which perform poorly,

and the higher the rate at which skilled workers must be produced to maintain balanced growth.

6.6 APPENDIX 1 - DERIVATION OF TABLE 1

Total differentiation of (6.3.1) and (6.3.2) gives the system of equations (evaluated at the optimum):

$$(6.6.1) \quad \begin{bmatrix} W^{\sigma\sigma} & W^{\sigma\alpha} \\ W^{\alpha\sigma} & W^{\alpha\alpha} \end{bmatrix} \begin{bmatrix} d\sigma \\ d\alpha \end{bmatrix} = \begin{bmatrix} -W^{\sigma s} & -W^{\sigma\zeta 1} & -W^{\sigma\zeta 2} & -W^{\sigma v} \\ -W^{\alpha s} & -W^{\alpha\zeta 1} & -W^{\alpha\zeta 2} & -W^{\alpha v} \end{bmatrix} \begin{bmatrix} ds \\ d\zeta_1 \\ d\zeta_2 \\ dv \end{bmatrix}$$

where, given sufficiently small f^{xy} , defining

$$\tau \equiv \frac{v}{\zeta} \quad \delta \equiv 1 - \frac{v}{s\zeta} \quad \epsilon \equiv 1 + \frac{v}{\zeta}$$

and noting that

$$\frac{\partial \tau}{\partial \alpha} = -\frac{v\pi^\alpha \Delta \zeta}{\zeta^2} < 0 \quad \text{and}$$

$$\frac{\partial^2 \tau}{\partial \alpha^2} = -\frac{v\Delta \zeta (\zeta \pi^{\alpha\alpha} - 2\pi^\alpha \pi^\alpha \Delta \zeta)}{\zeta^3} > 0$$

$$W^{\sigma\sigma} \approx \delta^2 f^{xx} + \epsilon^2 f^{yy} < 0$$

$$W^{\sigma\alpha} \approx -\frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s} f^x + f^y + \frac{1}{s} \alpha \right) - \frac{\tau}{s} - \sigma \frac{\partial \tau}{\partial \alpha} \left(\delta \frac{1}{s} f^{xx} - \epsilon f^{yy} \right)$$

$$= -\sigma \frac{\partial \tau}{\partial \alpha} \left(\delta \frac{1}{s} f^{XX} - \epsilon f^{YY} \right) \quad \text{using } W^\alpha = 0$$

which is relatively small by the second-order condition for a maximum.

$$W^{\sigma S} \approx \frac{\tau}{s^2} (f^X + \alpha + \delta \sigma f^{XX})$$

$$> 0 \quad \text{if } -\frac{f^{XX}}{f^X} \text{ is sufficiently small}$$

$$W^{\sigma \zeta 1} \approx \frac{\sigma(1-\pi)\tau}{\zeta} \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \delta f^{XX} - \epsilon f^{YY} \right) + \frac{\alpha(1-\pi)\tau}{s\zeta}$$

$$> 0 \quad \text{if } -\frac{f^{XX}}{f^X} \text{ is sufficiently small}$$

$$W^{\sigma \zeta 2} \approx \frac{\sigma \pi \tau}{\zeta} \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \delta f^{XX} - \epsilon f^{YY} \right) + \frac{\alpha \pi \tau}{s\zeta}$$

$$> 0 \quad \text{if } -\frac{f^{XX}}{f^X} \text{ is sufficiently small}$$

$$W^{\sigma V} \approx -\frac{1}{\zeta} \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \alpha \right) - \frac{\sigma}{\zeta} \left(\delta \frac{1}{s} f^{XX} - \epsilon f^{YY} \right)$$

$$< 0 \quad \text{if } -\frac{f^{XX}}{f^X} \text{ is sufficiently small}$$

$$W^{\alpha\alpha} \approx -\sigma \frac{\partial^2 \tau}{\partial \alpha^2} \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \alpha \right) - 2 \frac{\sigma}{s} \frac{\partial \tau}{\partial \alpha} + \left(\sigma \frac{\partial \tau}{\partial \alpha} \right)^2 \left(\frac{1}{s^2} f^{XX} + f^{YY} \right)$$

$$= -\sigma \frac{\pi^{\alpha\alpha}}{\pi} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s} f^X + f^Y + \alpha \right) + \left(\sigma \frac{\partial \tau}{\partial \alpha} \right)^2 \left(\frac{1}{s} f^{XX} + f^{YY} \right)$$

using $W^\alpha = 0$

< 0

$$W^{\alpha s} \approx \frac{\sigma}{s^2} \frac{\partial \tau}{\partial \alpha} \left(f^X + \alpha - \frac{\sigma \tau}{s} f^{XX} \right) + \frac{\sigma \tau}{s}$$

$$= \frac{\sigma}{s} \frac{\partial \tau}{\partial \alpha} \left(-f^Y + \frac{\sigma \tau}{s^2} f^{XX} \right) \quad \text{using } W^\alpha = 0$$

> 0

$$W^{\alpha\zeta} 1 \approx -\frac{\sigma \pi^\alpha \tau}{\zeta} \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \alpha \right) + \frac{2\sigma(1-\pi)}{\zeta} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \alpha \right)$$

$$+ \frac{\sigma(1-\pi)\tau}{s\zeta} - \frac{\sigma^2(1-\pi)\tau}{\zeta} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s^2} f^{XX} + f^{YY} \right)$$

$$= \sigma \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{\Delta \zeta} + \frac{1-\pi}{\zeta} \right) \left(\frac{1}{s} f^X + f^Y + \frac{1}{s} \alpha \right) - \frac{\sigma^2(1-\pi)\tau}{\zeta} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s^2} f^{XX} + f^{YY} \right)$$

using $W^\alpha = 0$

< 0

$$W^{\alpha\zeta} \approx -\sigma \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{\Delta \zeta} - \frac{\pi}{\zeta} \right) \left(\frac{1}{s} f^{XX} + f^Y + \frac{1}{s} \alpha \right) - \frac{\sigma^2 \pi \tau}{\zeta} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s^2} f^{XX} + f^{YY} \right)$$

using $W^\alpha = 0$

$$> 0 \quad \text{if } -\frac{f^{XX}}{f^X} \text{ and } -\frac{f^{YY}}{f^Y} \text{ are sufficiently small}^{12}$$

$$W^{\alpha V} \approx -\frac{\sigma}{V} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s} f^{XX} + f^Y + \frac{1}{s} \alpha \right) + \frac{\sigma \tau}{Vs} + \frac{\sigma^2}{\zeta} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s^2} f^{XX} + f^{YY} \right)$$

$$= \frac{\sigma^2}{\zeta} \frac{\partial \tau}{\partial \alpha} \left(\frac{1}{s^2} f^{XX} + f^{YY} \right) \quad \text{using } W^\alpha = 0$$

$$> 0$$

The second order conditions for a welfare-maximum require

$$\hat{W}^{\sigma\sigma} \hat{W}^{\alpha\alpha} - (\hat{W}^{\sigma\alpha})^2 > 0$$

Hence, it is required that $\hat{W}^{\sigma\alpha}$ is sufficiently small.

The results in Table 1 follow from the application of Cramer's rule to (6.6.1), and the properties of f and π .

6.7 APPENDIX 2 - DERIVATION OF TABLE 2

$$c_2 - c_1 = \frac{1}{\pi^\alpha}$$

Defining $\eta = -\frac{\pi^{\alpha\alpha}}{\pi^\alpha \pi^\alpha} > 0$ and using the results in Table 1 gives

the following response of the optimal penalty to parameter changes:

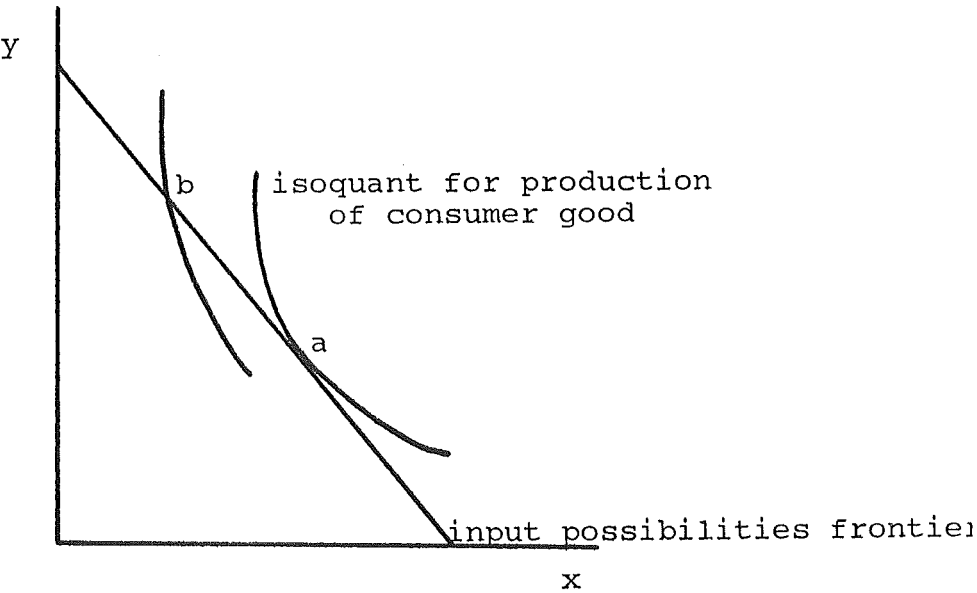
$$\frac{\partial c_2 - c_1}{\partial s} = \eta \frac{\partial \alpha}{\partial s} > 0$$

$$\frac{\partial c_2 - c_1}{\partial \zeta_1} = \eta \frac{\partial \alpha}{\partial \zeta_1} < 0$$

$$\frac{\partial c_2 - c_1}{\partial \zeta_2} = \eta \frac{\partial \alpha}{\partial \zeta_2} > 0$$

$$\frac{\partial c_2 - c_1}{\partial v} = \eta \frac{\partial \alpha}{\partial v} > 0$$

Figure 6.8.1



6.9 NOTES

1. The following notation is used:

$$f^X = \frac{\partial f}{\partial x} \quad \hat{f} = f(\hat{\cdot})$$

2. It is assumed that skilled workers are more productive at the margin than unskilled workers ($f^X > f^Y$), so that it always pays to use (un)skilled workers for (un)skilled tasks.

3. Unlike the model in Chapter 5, effort by students is not considered a variable. Thus, issues of moral hazard on the part of students are not relevant. Students should be thought of in terms of their characteristics such as "talented", "lazy", and so on.

4. To be distinguished from effort per educator which is measured by α .

5. $\frac{v}{s\zeta}$ is the number of educators needed to generate v skilled workers.

$\frac{1}{1 - \frac{v}{s\zeta}}$ is the number of new skilled workers needed to make available 1 skilled worker to the consumer good sector.

Hence, $\frac{v}{s\zeta} \frac{1}{1 - \frac{v}{s\zeta}} \alpha$ is the disutility from the effort of new

educators required to produce one skilled worker for the consumer good sector, so that the numerator in the left-hand

side of (6.3.3) measures the net welfare effect (from extra consumption and effort) from an additional skilled worker in the consumer good sector. Multiplying by the number of unskilled workers which, when released from the consumer good sector, generate a unit loss in welfare ($1/f^Y$), gives the required marginal rate of substitution.

6. On the left-hand side of (6.3.5), $\hat{\pi}^\alpha \hat{\Delta \zeta} / \hat{\zeta}$ measures the proportion of students who can be released due to a marginal increase in effort by all educators. Hence, per student, $\hat{\pi}^\alpha \hat{\Delta \zeta} / \hat{\zeta}$ unskilled workers and $\hat{\pi}^\alpha \hat{\Delta \zeta} / s \hat{\zeta}$ skilled workers can be released to the output sector. The numerator in the left-hand side therefore measures the output gain per student from the increase in effort. Further, 1 in the denominator measures the increase in effort per educator while $(\hat{\pi}^\alpha \hat{\Delta \zeta} / \hat{\zeta})^\alpha$ measures (on a per educator basis) the effort saved from a reduction in the number of educators. Multiplying $1 - (\hat{\pi}^\alpha \hat{\Delta \zeta} / \hat{\zeta})^\alpha$ by $1/s$ gives the net increase in effort on a per student basis. The left-hand side therefore measures the amount of output gained per unit of the additional effort; that is, it is the marginal rate of transforming effort into the consumer good by way of a release of labor from education to the consumer good sector.

7. Changes in the functional form of π , which also characterizes the technology in education, are not considered.

8. $\frac{\partial \hat{\sigma}}{\partial s} > 0$ etc.

9. These results require the additional assumption that the marginal returns to skilled and unskilled labor diminish relatively slowly ($-f^{xx}/f^x$ and $-f^{yy}/f^y$ are relatively small).

They also make use of the fact that f^{xy} is relatively small.

10. This condition is necessary and sufficient for $\hat{\alpha}$ to maximize the educator's expected utility.

11. For instance, if $\pi(\alpha) = \alpha^{\frac{1}{2}}$ $\alpha \in (0,1)$, then $\alpha \leq \frac{1}{4}$ suffices.

$$12. \quad \zeta = \pi \Delta \zeta + \zeta_1 > \pi \Delta \zeta \quad \Rightarrow \quad \frac{1}{\Delta \zeta} - \frac{\pi}{\zeta} > 0$$

CHAPTER 7

SUMMARY, CONCLUDING REMARKS AND SUGGESTIONS
FOR FURTHER RESEARCH

This thesis has investigated issues of moral hazard in a variety of contexts not yet fully considered in the principal-agent literature.

Chapter 2 uses a Slutsky-like equation to derive properties of principal-agent contracts in a conventional single agent model. It shows that availability of data on the responsiveness of contracts to disturbances in the agent's opportunity cost allows predictions about the responsiveness of those contracts to disturbances in the outcomes of the principal-agent relationship. These predictions depend on whether moral hazard is present.

The analysis is only a first step in the use of Slutsky equations in principal-agent theory. While results are given for any number of states for a model with enforceable actions, the results for the same model with unenforceable actions are limited by the assumption, that only two outcomes are possible. Further, the model is a very simple one, involving a risk-neutral principal, a single agent and a single time-period. Slutsky-like equations should be available also for more complicated formulations.

Chapter 3 gives a characterization of contracts when the principal can choose an action affecting the expected outcome, and perceives that the action can be used to motivate the agent. The action therefore has incentive effects, as well as direct productivity effects, and, as such, differs from existing analyses which allow the principal to choose an action.

In some situations the agent may control his own and also the principal's action. In this case the principal has to trade one form of moral hazard (an undesirable action on the part of the agent) against another (a requirement that the principal undertake an action that is too costly). The results show, that the tradeoff involves the costliest action on the part of the principal, in order to avoid the least desirable action by the agent.

In a broader sense, allowing the principal to choose actions eliminates one of the asymmetries characteristic of most principal-agent models. Chapter 4 eliminates another typical asymmetry, namely the assumption that the principal alone determines the terms of compensation. When the principal chooses an action, and agents bargain for a share of the outcome (so that the distinction between principal and agent disappears), the model becomes a natural extension of a commodity exchange model with uncertain endowments. The extension involves the dependence of the probability

distribution over endowments on the actions of the agents.

The chapter addresses the question of what contracts might be observed as a result of bargaining among agents, using the core as the solution concept. The analysis considers a class of economies in which coalitions may be confronted with moral hazard on the part of other coalitions. It is shown that the core exists, implying that instead of multiple coalitions, the economy should feature collective agreements about the optimal set of actions. The precise form of a contract implementing those actions is also given. It is further shown that the core satisfies the property that identical agents are treated equally, and that the core shrinks to a competitive contract as the economy grows large.

The model leaves room for extension on several fronts. The results make use of a number of restrictions on the set of technically feasible actions, the probability distribution over endowments, and the agents' characteristics. They are a sufficient set of restrictions - whether they are also necessary is yet to be determined. Further, the extent of moral hazard is limited by an assumption, that moral hazard is not a problem within coalitions. An immediate, although apparently difficult extension, would allow moral hazard within as well as across coalitions.

Chapters 5 and 6 address moral hazard problems in education. In chapter 5, government provision of student

loans, with an implicit insurance policy against low future earnings, gives students an incentive to shirk. Since this lowers the government's ability to fund future education, society has an interest in preventing such behavior. The analysis derives optimal levels of insurance when the probability of graduating depends only on students' effort, and when that probability also depends on the length of education. Also considered is the effect of changes in population growth on optimal levels of insurance, effort and skill.

The contribution of the model to principal-agent theory is that optimal levels of insurance are derived in a model in which the length of the principal-agent relationship (namely, the length of education, during which the government can influence students' actions) is endogenous.

The model is limited by the fact that the loan is exogenously given. An immediate extension would make the loan a choice variable of the government.

In Chapter 6 the government hires educators to train students. In order to reduce shirking, the government penalizes educators whose classes perform poorly. At issue are the characterization of the optimal contract, and the optimal adjustment of the penalty to exogenous disturbances.

The significance of the analysis for principal-agent theory derives from the fact that the model describes a general equilibrium. It determines the proportion of educators (who

act as agents to students) in the workforce, as well as educators' opportunity cost. In the literature these have generally been treated as given.

The model considers only one aspect of education policy, namely the nature of penalties in contracts offered to educators. Other issues, such as the properties of tuition fees payable by students, and the extent of insurance in education, if individuals are risk-averse have yet to be considered.

BIBLIOGRAPHY

- Alchian, A. and Demsetz, H. Production, information costs and economic organization. American Economic Review, 62: 777-95, 1972.
- Arnott, R. and Stiglitz, J. The basic analytics of moral hazard. Scandinavian Journal of Economics, 90: 383-413, 1988.
- Arrow, K. Uncertainty and the welfare economics of medical care. American Economic Review, 53: 941-73, 1963.
- Arrow, K. Essays in the theory of risk bearing. Amsterdam, North Holland, 1971.
- Arrow, K. The economics of agency. In Pratt, J. and Zeckhauser, R. (eds.) Principals and agents: The structure of business. Boston, Harvard Business School Press, 1985. 241p.
- Arrow, K. The economics of moral hazard: further comment. American Economic Review, 58: 537-8, 1968.
- Arrow, K. Uncertainty and the welfare economics of medical care. American Economics Review, 53: 941-73, 1963.
- Becker, G. Investment in human capital: A theoretical analysis. Journal of Political Economy, 70: 9-49, 1962.
- Ben-Porath, Y. The production of human capital and the life cycle of earnings. Journal of Political Economy, 75: 352-65, 1967.

- Bernheim, B. and Whinston, M. Common agency. Econometrica, 54: 923-43, 1986.
- Bernheim, B. and Whinston, M. Coalition-proof Nash equilibria - II. Applications. Journal of Economic Theory, 42: 13-29, 1987a.
- Bernheim, B. and others. Coalition-proof Nash equilibria - I. Concepts. Journal of Economic Theory, 42: 1-12, 1987b.
- Bhattacharya, S. and Guasch, J. Heterogeneity, tournaments and hierarchies. Journal of Political Economy, 96: 867-81, 1988.
- Borch, K. Equilibrium in a reinsurance market. Econometrica, 30: 424-44, 1962.
- Brown, M. and others. A new class of sufficient conditions for the first-order approach to principal-agent problems. Economics Letters, 21: 1-6, 1986.
- Cantor, R. Work effort and contract length. Economica, 55: 343-53, 1988.
- Carmichael, H. The agent-agents problem: payment by relative output. Journal of Labor Economics, 1: 50-65, 1983.
- Chichilnisky, G. and Kalman, P. Properties of critical points and operators in economics. Journal of Mathematical Analysis and Applications, 57: 340-9, 1977.
- Chichilnisky, G. and Kalman, P. Comparative statics of less neoclassical agents. International Economic Review, 19: 141-8, 1978.
- Cooper, R. and Ross, T. Product warranties and double moral

- hazard. Rand Journal of Economics, 16:103-13, 1985.
- Dasgupta, P. and others. The implementation of social choice rules: Some general results on incentive compatibility. Review of Economic Studies, 46: 185-216, 1979.
- Debreu, G. Theory of value. New York, Wiley, 1959.
- Debreu, G. and Scarf, H. A limit theorem on the core of an economy. International Economic Review, 4: 235-46, 1963.
- Demski, J. and Sappington, D. Optimal incentive contracts with multiple agents. Journal of Economic Theory, 33: 152-71, 1984.
- Eaton, J. and Rosen, H. Taxation, human capital, and uncertainty. American Economic Review, 70:705-15, 1980.
- Edgeworth, F. Mathematical Psychics. London, Paul Kegan, 1881. 150p.
- Emons, W. Warranties, moral hazard, and the lemons problem. Journal of Economic Theory, 46: 16-33, 1988.
- Fellingham, J. and others. Contracts without memory in multiperiod agency models. Journal of Economic Theory, 37: 340-55, 1985.
- Foley, D. Lindahl's solution and the core of an economy with public goods. Econometrica, 38: 66-72, 1970.
- Gjesdal, F. Information and incentives: The agency information problem. Review of Economic Studies, 64: 373-90, 1982.
- Gjesdal, F. Piecewise linear incentive schemes. Scandinavian Journal of Economics, 90: 305-28, 1988.

- Green, J. and Stokey, N. A comparison of tournaments and contracts. Journal of Political Economy, 91: 349-64, 1983.
- Grossman, S. and Hart, O. An analysis of the principal-agent problem. Econometrica, 51: 7-45, 1983.
- Harris, M. and Raviv A. Some results on incentive contracts with applications to education and employment, health insurance, and law enforcement. American Economic Review, 68: 20-30, 1978.
- Harris, M. and Raviv, A. Optimal incentive contracts with imperfect information. Journal of Economic Theory, 20: 231-59, 1979.
- Hart, O. The market mechanism as an incentive scheme. Bell Journal of Economics, 14: 366-82, 1983.
- Hildenbrand, W. Core and equilibria of a large economy. Princeton, Princeton University Press, 1974. 251p.
- Hirschleifer, J. and Riley, J. The analytics of uncertainty and information: An expository survey. Journal of Economic Literature, 17: 1375-421, 1979.
- Holmström, B. Moral hazard and observability. Bell Journal of Economics, 10: 74-91, 1979.
- Holmström, B. Moral hazard in teams. Bell Journal of Economics, 13: 324-40, 1982.
- Hu, S. C. Education and economic growth. Review of Economic Studies, 34: 19-30, 1976.
- Hurwicz, L. On informationally decentralized systems. In

- McGuire, C. and Radner, R. (eds.) Decision and organization. Amsterdam, North Holland, 1972.
- Hurwicz, L. and Shapiro, L. Incentive structures maximizing residual gain under incomplete information. Bell Journal of Economics, 9:180-91, 1978.
- Intriligator, M. Mathematical organization and economic theory. Englewood Cliffs, Pentice-Hall, 1971. 508p.
- Jensen, M. and Meckling, W. Theory of the firm: managerial behavior, agency costs and ownership structure. Journal of Financial Economics, 3: 305-60, 1976.
- Jewitt, I. Justifying the first-order approach to principal-agent problems. Econometrica, 56:1177-90, 1988.
- Kalman, P. and Intriligator, M. Generalized comparative statics with applications to consumer theory and producer theory. International Economic Review, 14: 473-86, 1973.
- Kemp, M. and others. Three pitfalls in the construction of family- based models of population growth. European Economic Review, 25: 345-54, 1984.
- Kihlstrom, R. The role of insurance in the allocation of risk. American Economic Review, 61: 371-9, 1971.
- Laffont, J-J. The economics of uncertainty and information. Cambridge, MIT Press, 1989. 289p.
- Lazaer, E. and Rosen, S. Rank-order tournaments as optimum labour contracts. Journal of Political Economy, 89: 841- 64, 1981.

- Levhari, D. and Weiss, Y. The effect of risk on the investment in human capital. American Economic Review, 64: 950-63, 1974.
- McAfee, P. and McMillan, J. Bidding for contracts: A principal-agent analysis. Rand Journal of Economics, 17: 326-38, 1986.
- MacDonald, G. New directions in the conomic theory of agency Canadian Journal of Economics, 3: 415-40, 1984.
- MacGuire, R. and Ohsfeldt, R. Self-interest, agency theory and political voting behavior: The ratification of the US constitution. American Economic Review, 79: 219-34, 1989.
- Malcomson, J. Work incentives, hierarchy and internal labour markets. Journal of Political Economy, 92: 486-507, 1984.
- Malcomson, J. Rank-order contracts for a principal with many agents. Review of Economic Studies, 53: 807-17, 1986.
- Manning, R. Optimal aggregative development of a skilled work force. Quarterly Journal of Economics, 89: 505-11, 1975.
- Manning, R. Issues in optimal educational policy in the context of balanced growth. Journal of Economic Theory, 13: 380-95, 1976.
- Manning, R. Some issues in human capital theory. Paper presented at the 14th Conference of Economists. Sidney, University of New South Wales, 1985.
- Milgrom, P. Good news and bad news: representation theorems and applications. Bell Journal of Economics, 12:380-91, 1981.
- Milleron, J-C. Theory of value with public goods: a survey

- article. Journal of Economic Theory, 5: 419-77, 1972.
- Mirrlees, J. An exploration in the theory of optimum income taxation. Review of Economic Studies, 38: 175-208, 1971.
- Mirrlees, J. The theory of moral hazard and unobservable behavior, Part I. Oxford, Nuffield College, 1975. (Mimeo)
- Mirrlees, J. The optimal structure of incentives and authority within an organization. Bell Journal of Economics, 7: 105-31, 1976.
- Mookherjee, D. Optimal incentive schemes with many agents. Review of Economic Studies, 51: 433-46, 1984.
- Muench, T. The core and the Lindahl equilibrium of an economy with a public good: an example. Journal of Economic Theory, 4: 241-55, 1972.
- Myerson, R. Incentive compatibility and the bargaining problem. Econometrica, 47: 61-73, 1979.
- Nalebuff, B. and Stiglitz, J. Prizes and incentives: towards a general theory of compensation and competition. Bell Journal of Economics, 14: 21-43, 1983.
- Nerlove, M. and others. Investment in human and nonhuman capital, transfers among siblings, and the role of government. Econometrica, 52: 1191-8, 1984.
- Palfrey, T. and Srivastava, S. Private information in large economies. Journal of Economic Theory, 39: 34-58, 1986.
- Palfrey, T. and Srivastava, S. Implementation with incomplete information in exchange economies. Econometrica, 57: 115-

34, 1989.

Pauly, M. Clubs, commonality, and the core: an integration of game theory and the theory of public goods. Economica, 34: 314-24, 1967.

Pauly, M. The economics of moral hazard: a comment. American Economic Review, 58: 531-7, 1968.

Pauly, M. Overinsurance and public provision of insurance: the roles of moral hazard and adverse selection. Quarterly Journal of Economics, 88: 44-62, 1974.

Postlewaite, A. and Schmeidler, D. Implementation in differential information economies. Journal of Economic Theory, 39: 14-33, 1986.

Radner, R. Monitoring cooperative agreements in a repeated principal-agent relationship. Econometrica, 49: 1127-48, 1981.

Radner, R. Repeated partnership games with imperfect monitoring and no discounting. Review of Economic Studies, 53: 43-57, 1986.

Razin, A. Investment in human capital and economic growth Metroeconomica, 34: 104-15, 1972a.

Razin, A. Optimum investment in human capital. Review of Economic Studies, 39: 455-60, 1972b.

Richter, D. The core of a public goods economy. International Economic Review, 15: 131-42, 1974.

Rogerson, W. The first-order approach to principal-agent

- problems. Econometrica, 56: 1357-67, 1985.
- Rosen, S. Prizes and incentives in elimination tournaments. American Economic Review, 76: 701-15, 1986.
- Rosenthal, R. External economies and cores. Journal of Economic Theory, 3: 182-8, 1971.
- Ross, S. The economic theory of agency: The principal's problem. American Economic Review, 63: 134-9, 1973.
- Samuelson, P. An exact consumption-loan model with or without the social contrivance of money. Journal of Political Economy, 66: 467-82, 1958.
- Sappington, D. Incentive contracting with asymmetric and imperfect precontractual knowledge. Journal of Economic Theory, 34: 52-70, 1984.
- Schultz, T. Investment in human capital. American Economic Review, 51: 1-17, 1961.
- Shapley, L. and Shubik, M. On the core of an economic system with externalities. American Economic Review, 678-84, 1969.
- Shavell, S. On moral hazard and insurance. Quarterly Journal of Economics, 93: 541-62, 1979a.
- Shavell, S. Risk sharing and incentives in the principal and agent relationship. Bell Journal of Economics, 10: 55-73, 1979b.
- Shavell, S. On liability and insurance. Bell Journal of Economics, 13: 120-32, 1982.
- Spence, M. and Zeckhauser, R. Insurance, information and

individual action. American Economic Review, 61: 380-91, 1971.

Stiglitz, J. Incentives and risk-sharing in sharecropping. Review of Economic Studies, 42: 219-55, 1974.

Stiglitz, J. Incentives, risk and information: notes towards a theory of hierarchy. Bell Journal of Economics, 6: 552-79, 1975.

Stiglitz, J. and Weiss, A. Credit rationing in markets with imperfect information. American Economic Review, 393-410, 1981.

Telser, L. Voting and paying for public goods: an application of the theory of the core. Journal of Economic Theory, 27: 376-409, 1982.

Townsend, R. Optimal contracts and competitive markets with costly state verification. Journal of Economic Theory, 21: 265-93, 1979.

Varian, H. Microeconomic analysis. 2nd ed. New York, Norton, 1984. 348p.

Weisbrod, B. Education and investment in human capital. Journal of Political Economy, 70: 106-23, 1962.

Weitzman, M. Efficient incentive contracts. Quarterly Journal of Economics, 94: 719-30, 1980.

Wooders, M. Equilibria, the core, and jurisdiction structures in economies with a local public good. Journal of Economic Theory, 18: 328-48, 1978.

Wooders, M. Equilibria, the core and jurisdiction structures in economies with a local public good: a correction.

Journal of Economic Theory, 25: 144-51, 1981.

Zeckhauser, R. Medical insurance: a case study of the tradeoff between risk spreading and appropriate incentives. Journal of Economic Theory, 2: 10-26, 1970.